Objectives

Section		You should be able to:	Review Exercises
2.1	1	Convert between decimals and degrees, minutes, seconds forms for angles (p. 96)	86
	2	Find the arc length of a circle (p. 97)	87, 88
	3	Convert from degrees to radians and from radians to degrees (p. 98)	1–8
	4	Find the area of a sector of a circle (p. 101)	87
	5	Find the linear speed of an object traveling in circular motion (p. 102)	89–92
2.2	1	Find the exact values of the trigonometric functions using a point on the unit circle (p. 109)	83, 97
	2	Find the exact values of the trigonometric functions of quadrantal angles (p. 111)	10, 17, 18, 20, 97
	3	Find the exact values of the trigonometric functions of $\frac{\pi}{4} = 45°$ (p. 113)	9, 11, 13, 15, 16
	4	Find the exact values of the trigonometric functions of $\frac{\pi}{6} = 30°$ and $\frac{\pi}{3} = 60°$ (p. 114)	9–15
	5	Find the exact values of the trigonometric functions for integer multiples of $\frac{\pi}{6} = 30°$, $\frac{\pi}{4} = 45°$, and $\frac{\pi}{3} = 60°$ (p. 116)	13–16, 19, 97
	6	Use a calculator to approximate the value of a trigonometric function (p. 117)	79, 80
	7	Use circle of radius r to evaluate the trigonometric functions (p. 118)	84
2.3	1	Determine the domain and the range of the trigonometric functions (p. 123)	85
	2	Determine the period of the trigonometric functions (p. 125)	85
	3	Determine the signs of the trigonometric functions in a given quadrant (p. 127)	81, 82
	4	Find the values of the trigonometric functions using fundamental identities (p. 127)	21–30
	5	Find the exact values of the trigonometric functions of an angle given one of the functions and the quadrant of the angle (p. 130)	31–46
	6	Use even–odd properties to find the exact values of the trigonometric functions (p. 132)	27–30
2.4	1	Graph functions of the form $y = A\sin(\omega x)$ using transformations (p. 138)	47
	2	Graph functions of the form $y = A\cos(\omega x)$ using transformations (p. 139)	48
	3	Determine the amplitude and period of sinusoidal functions (p. 140)	63–68
	4	Graph sinusoidal functions using key points (p. 142)	47, 48, 67, 68, 93
	5	Find an equation for a sinusoidal graph (p. 145)	75–78
2.5	1	Graph functions of the form $y = A\tan(\omega x) + B$ and $y = A\cot(\omega x) + B$ (p. 153)	53, 54, 56
	2	Graph functions of the form $y = A\csc(\omega x) + B$ and $y = A\sec(\omega x) + B$ (p. 155)	57
2.6	1	Graph sinusoidal functions of the form $y = A\sin(\omega x - \phi) + B$ (p. 158)	49, 50, 59, 60, 69–74, 94
	2	Find a sinusoidal function from data (p. 162)	95, 96

Review Exercises

In Problems 1–4, convert each angle in degrees to radians. Express your answer as a multiple of π.

1. 135° **2.** 210° **3.** 18° **4.** 15°

In Problems 5–8, convert each angle in radians to degrees.

5. $\frac{3\pi}{4}$ **6.** $\frac{2\pi}{3}$ **7.** $-\frac{5\pi}{2}$ **8.** $-\frac{3\pi}{2}$

In Problems 9–30, find the exact value of each expression. Do not use a calculator.

9. $\tan\frac{\pi}{4} - \sin\frac{\pi}{6}$ **10.** $\cos\frac{\pi}{3} + \sin\frac{\pi}{2}$ **11.** $3\sin 45° - 4\tan\frac{\pi}{6}$

12. $4\cos 60° + 3\tan\frac{\pi}{3}$ **13.** $6\cos\frac{3\pi}{4} + 2\tan\left(-\frac{\pi}{3}\right)$ **14.** $3\sin\frac{2\pi}{3} - 4\cos\frac{5\pi}{2}$

15. $\sec\left(-\frac{\pi}{3}\right) - \cot\left(-\frac{5\pi}{4}\right)$

16. $4\csc\frac{3\pi}{4} - \cot\left(-\frac{\pi}{4}\right)$

17. $\tan\pi + \sin\pi$

18. $\cos\frac{\pi}{2} - \csc\left(-\frac{\pi}{2}\right)$

19. $\cos 540° - \tan(-405°)$

20. $\sin 270° + \cos(-180°)$

21. $\sin^2 20° + \frac{1}{\sec^2 20°}$

22. $\frac{1}{\cos^2 40°} - \frac{1}{\cot^2 40°}$

23. $\sec 50° \cdot \cos 50°$

24. $\tan 10° \cdot \cot 10°$

25. $\sin 50° \cdot \csc 410°$

26. $\tan 20° \cdot \cos(-20°) \cdot \csc 20°$

27. $\sin(-40°) \cdot \csc 40°$

28. $\tan(-20°) \cdot \cot 20°$

29. $\sin 405° \cdot \sec(-45°)$

30. $\cos 250° \cdot \sec(-70°)$

In Problems 31–46, find the exact value of each of the remaining trigonometric functions.

31. $\sin\theta = \frac{4}{5}, \quad 0 < \theta < \frac{\pi}{2}$

32. $\tan\theta = \frac{1}{4}, \quad 0 < \theta < \frac{\pi}{2}$

33. $\tan\theta = \frac{12}{5}, \quad \sin\theta < 0$

34. $\cot\theta = \frac{12}{5}, \quad \cos\theta < 0$

35. $\sec\theta = -\frac{5}{4}, \quad \tan\theta < 0$

36. $\csc\theta = -\frac{5}{3}, \quad \cot\theta < 0$

37. $\sin\theta = \frac{12}{13}, \quad \theta$ in quadrant II

38. $\cos\theta = -\frac{3}{5}, \quad \theta$ in quadrant III

39. $\sin\theta = -\frac{5}{13}, \quad \frac{3\pi}{2} < \theta < 2\pi$

40. $\cos\theta = \frac{12}{13}, \quad \frac{3\pi}{2} < \theta < 2\pi$

41. $\tan\theta = \frac{1}{3}, \quad 180° < \theta < 270°$

42. $\tan\theta = -\frac{2}{3}, \quad 90° < \theta < 180°$

43. $\sec\theta = 3, \quad \frac{3\pi}{2} < \theta < 2\pi$

44. $\csc\theta = -4, \quad \pi < \theta < \frac{3\pi}{2}$

45. $\cot\theta = -2, \quad \frac{\pi}{2} < \theta < \pi$

46. $\tan\theta = -2, \quad \frac{3\pi}{2} < \theta < 2\pi$

In Problems 47–62, graph each function. Each graph should contain at least two periods.

47. $y = 2\sin(4x)$

48. $y = -3\cos(2x)$

49. $y = -2\cos\left(x + \frac{\pi}{2}\right)$

50. $y = 3\sin(x - \pi)$

51. $y = \tan(x + \pi)$

52. $y = -\tan\left(x - \frac{\pi}{2}\right)$

53. $y = -2\tan(3x)$

54. $y = 4\tan(2x)$

55. $y = \cot\left(x + \frac{\pi}{4}\right)$

56. $y = -4\cot(2x)$

57. $y = 4\sec(2x)$

58. $y = \csc\left(x + \frac{\pi}{4}\right)$

59. $y = 4\sin(2x + 4) - 2$

60. $y = 3\cos(4x + 2) + 1$

61. $y = 4\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)$

62. $y = 5\cot\left(\frac{x}{3} - \frac{\pi}{4}\right)$

In Problems 63–66, determine the amplitude and period of each function without graphing.

63. $y = 4\cos x$

64. $y = \sin(2x)$

65. $y = -8\sin\left(\frac{\pi}{2}x\right)$

66. $y = -2\cos(3\pi x)$

In Problems 67–74, find the amplitude, period, and phase shift of each function. Graph each function. Show at least two periods.

67. $y = 4\sin(3x)$

68. $y = 2\cos\left(\frac{1}{3}x\right)$

69. $y = 2\sin(2x - \pi)$

70. $y = -\cos\left(\frac{1}{2}x + \frac{\pi}{2}\right)$

71. $y = \frac{1}{2}\sin\left(\frac{3}{2}x - \pi\right)$

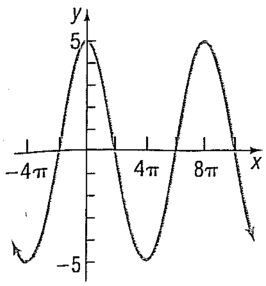
72. $y = \frac{3}{2}\cos(6x + 3\pi)$

73. $y = -\frac{2}{3}\cos(\pi x - 6)$

74. $y = -7\sin\left(\frac{\pi}{3}x + \frac{4}{3}\right)$

In Problems 75–78, find a function whose graph is given.

75.



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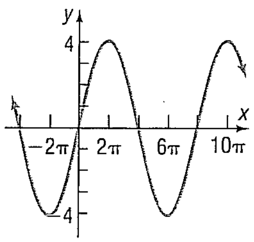

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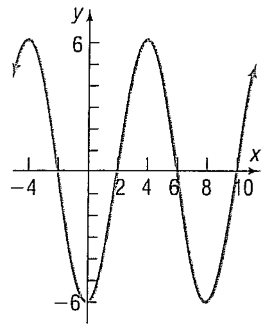

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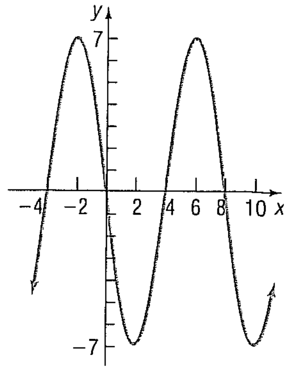


79. Use a calculator to approximate $\sin \frac{\pi}{8}$. Round the answer to two decimal places.

80. Use a calculator to approximate $\sec 10°$. Round the answer to two decimal places.

81. Determine the signs of the six trigonometric functions of an angle θ whose terminal side is in quadrant III.

82. Name the quadrant θ lies in if $\cos \theta > 0$ and $\tan \theta < 0$.

83. Find the exact values of the six trigonometric functions of t if $P = \left(-\frac{1}{3}, \frac{2\sqrt{2}}{3}\right)$ is the point on the unit circle that corresponds to t.

84. Find the exact value of $\sin t$, $\cos t$, and $\tan t$ if $P = (-2, 5)$ is the point on the circle that corresponds to t.

85. What is the domain and the range of the secant function? What is the period?

86. (a) Convert the angle $32°20'35''$ to a decimal in degrees. Round the answer to two decimal places.
(b) Convert the angle $63.18°$ to D°M′S″ form. Express the answer to the nearest second.

87. Find the length of the arc subtended by a central angle of 30° on a circle of radius 2 feet. What is the area of the sector?

88. The minute hand of a clock is 8 inches long. How far does the tip of the minute hand move in 30 minutes? How far does it move in 20 minutes?

89. Angular Speed of a Race Car A race car is driven around a circular track at a constant speed of 180 miles per hour. If the diameter of the track is $\frac{1}{2}$ mile, what is the angular speed of the car? Express your answer in revolutions per hour (which is equivalent to laps per hour).

90. Merry-Go-Rounds A neighborhood carnival has a merry-go-round whose radius is 25 feet. If the time for one revolution is 30 seconds, how fast is the merry-go-round going?

91. Lighthouse Beacons The Montauk Point Lighthouse on Long Island has dual beams (two light sources opposite each other). Ships at sea observe a blinking light every 5 seconds. What rotation speed is required to do this?

92. Spin Balancing Tires The radius of each wheel of a car is 16 inches. At how many revolutions per minute should a spin balancer be set to balance the tires at a speed of 90 miles per hour? Is the setting different for a wheel of radius 14 inches? If so, what is this setting?

93. Alternating Voltage The electromotive force E, in volts, in a certain ac (alternating circuit) circuit obeys the function

$$E(t) = 120 \sin(120\pi t), \qquad t \geq 0$$

where t is measured in seconds.
(a) What is the maximum value of E?
(b) What is the period?
(c) Graph this function over two periods.

94. Alternating Current The current I, in amperes, flowing through an ac (alternating current) circuit at time t is

$$I(t) = 220 \sin\left(30\pi t + \frac{\pi}{6}\right), \qquad t \geq 0$$

(a) What is the period?
(b) What is the amplitude?
(c) What is the phase shift?
(d) Graph this function over two periods.

95. Monthly Temperature The following data represent the average monthly temperatures for Phoenix, Arizona.

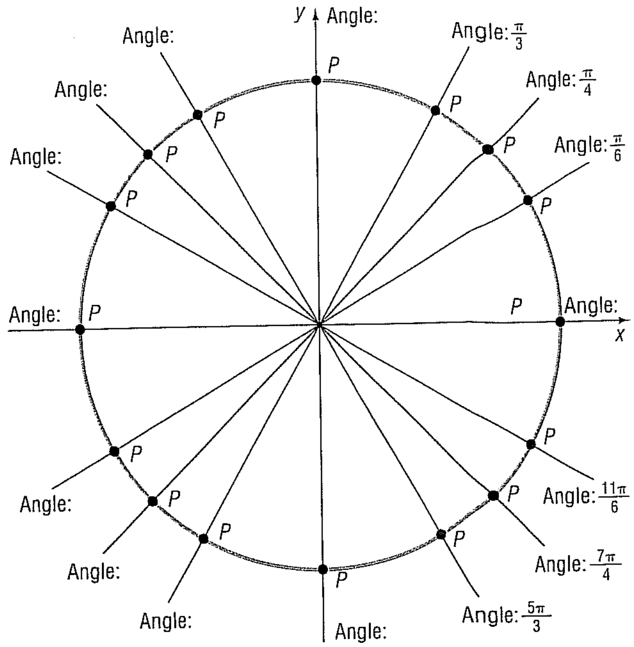
Month, m	Average Monthly Temperature, T
January, 1	51
February, 2	55
March, 3	63
April, 4	67
May, 5	77
June, 6	86
July, 7	90
August, 8	90
September, 9	84
October, 10	71
November, 11	59
December, 12	52

SOURCE: U.S. National Oceanic and Atmospheric Administration

(a) Draw a scatter diagram of the data for one period.
(b) Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that fits the data.
(c) Draw the sinusoidal function found in part (b) on the scatter diagram.
(d) Use a graphing utility to find the sinusoidal function of best fit.
(e) Graph the sinusoidal function of best fit on the scatter diagram.

96. Hours of Daylight According to the *Old Farmer's Almanac,* in Las Vegas, Nevada, the number of hours of sunlight on the summer solstice is 14.63 and the number of hours of sunlight on the winter solstice is 9.72.
(a) Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that fits the data.
(b) Use the function found in part (a) to predict the number of hours of sunlight on April 1, the 91st day of the year.
(c) Draw a graph of the function found in part (a).
(d) Look up the number of hours of sunlight for April 1 in the *Old Farmer's Almanac* and compare the actual hours of daylight to the results found in part (c).

97. Unit Circle On the unit circle below, fill in the missing angles (in radians) and the corresponding terminal points P of each angle.



CHAPTER TEST

In Problems 1–3, convert each angle in degrees to radians. Express your answer as a multiple of π.

1. $260°$ **2.** $-400°$ **3.** $13°$

In Problems 4–6 convert each angle in radians to degrees.

4. $-\frac{\pi}{8}$ **5.** $\frac{9\pi}{2}$ **6.** $\frac{3\pi}{4}$

In Problems 7–12, find the exact value of each expression.

7. $\sin\frac{\pi}{6}$ **8.** $\cos\left(-\frac{5\pi}{4}\right) - \cos\frac{3\pi}{4}$

9. $\cos(-120°)$ **10.** $\tan 330°$

11. $\sin\frac{\pi}{2} - \tan\frac{19\pi}{4}$ **12.** $2\sin^2 60° - 3\cos 45°$

In Problems 13–16, use a calculator to evaluate each expression. Round your answer to three decimal places.

13. $\sin 17°$ **14.** $\cos\frac{2\pi}{5}$ **15.** $\sec 229°$ **16.** $\cot\frac{28\pi}{9}$

17. Fill in each table entry with the sign of each function.

	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\sec\theta$	$\csc\theta$	$\cot\theta$
θ in QI						
θ in QII						
θ in QIII						
θ in QIV						

18. If $f(x) = \sin x$ and $f(a) = \frac{3}{5}$, find $f(-a)$.

In Problems 19–21 find the value of the remaining five trigonometric functions of θ.

19. $\sin\theta = \frac{5}{7}$, θ in quadrant II

20. $\cos\theta = \frac{2}{3}$, $\frac{3\pi}{2} < \theta < 2\pi$

21. $\tan\theta = -\frac{12}{5}$, $\frac{\pi}{2} < \theta < \pi$

In Problems 22–24, the point (x, y) is on the terminal side of angle θ in standard position. Find the exact value of the given trigonometric function.

22. $(2, 7)$, $\sin\theta$ **23.** $(-5, 11)$, $\cos\theta$

24. $(6, -3)$, $\tan\theta$

In Problems 25 and 26, graph the function.

25. $y = 2\sin\left(\frac{x}{3} - \frac{\pi}{6}\right)$ **26.** $y = \tan\left(-x + \frac{\pi}{4}\right) + 2$

27. Write an equation for a sinusoidal graph with the following properties:

$A = -3$ period $= \frac{2\pi}{3}$ phase shift $= -\frac{\pi}{4}$

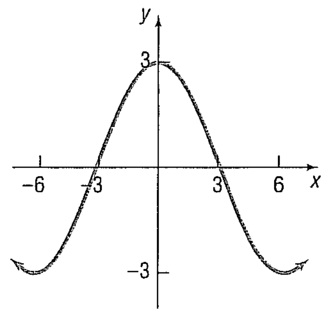
28. Logan has a garden in the shape of a sector of a circle; the outer rim of the garden is 25 feet long and the central angle of the sector is 50°. She wants to add a 3-foot wide walk to the outer rim; how many square feet of paving blocks will she need to build the walk?

29. Hungarian Adrian Annus won the gold medal for the hammer throw at the 2004 Olympics in Athens with a winning distance of 83.19 meters.* The event consists of swinging a 16-pound weight attached to a wire 190 centimeters long—in a circle and then releasing it. Assuming his release is at a 45° angle to the ground, the hammer will travel a distance of $\frac{v_0^2}{g}$ meters, where $g = 9.8$ meters/second^2 and v_0 is the linear speed of the hammer when released. At what rate (rpm) was he swinging the hammer upon release?

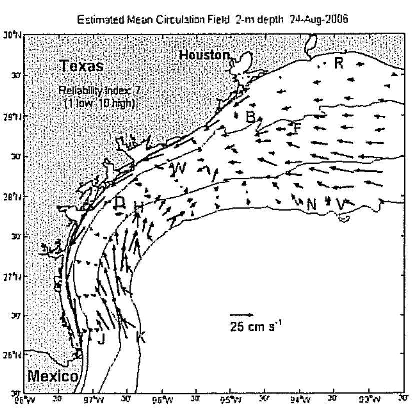
* Annus was stripped of his medal after refusing to cooperate with postmedal drug testing.

CUMULATIVE REVIEW

1. Find the real solutions, if any, of the equation $2x^2 + x - 1 = 0$.
2. Find an equation for the line with slope -3 containing the point $(-2, 5)$.
3. Find an equation for a circle of radius 4 and center at the point $(0, -2)$.
4. Discuss the equation $2x - 3y = 12$. Graph it.
5. Discuss the equation $x^2 + y^2 - 2x + 4y - 4 = 0$. Graph it.
6. Use transformations to graph the function $y = (x - 3)^2 + 2$.
7. Sketch a graph of each of the following functions. Label at least three points on each graph.
   (a) $y = x^2$ (b) $y = x^3$
   (c) $y = \sin x$ (d) $y = \tan x$
8. Find the inverse function of $f(x) = 3x - 2$.
9. Find the exact value of $(\sin 14°)^2 + (\cos 14°)^2 - 3$.
10. Graph $y = 3\sin(2x)$.
11. Find the exact value of $\tan\frac{\pi}{4} - 3\cos\frac{\pi}{6} + \csc\frac{\pi}{6}$.
12. Find a sinusoidal function for the following graph.



CHAPTER PROJECTS



I. Tides The given table is a partial tide table for November 2006 for the Sabine Bank Lighthouse, a shoal located offshore from Texas where the Sabine River empties into the Gulf of Mexico.

1. On November 15, when was the tide high? This is called *high tide*. On November 19, when was the tide low? This is called *low tide*. Most days will have two high tides and two low tides.