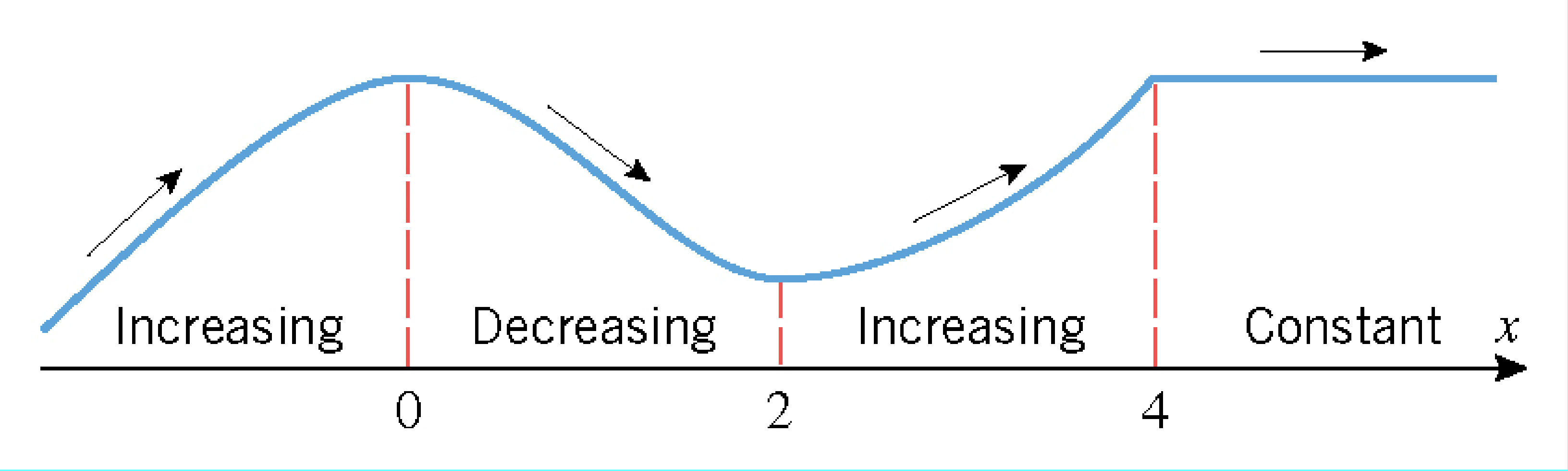
The Derivative in Graphing

Increasing and Decreasing Functions

Increasing: If $x_1 < x_2$, then $f(x_1) < f(x_2)$, f(x) is increasing.
If $f'(x) > 0$ on the interval, f(x) is increasing on that interval

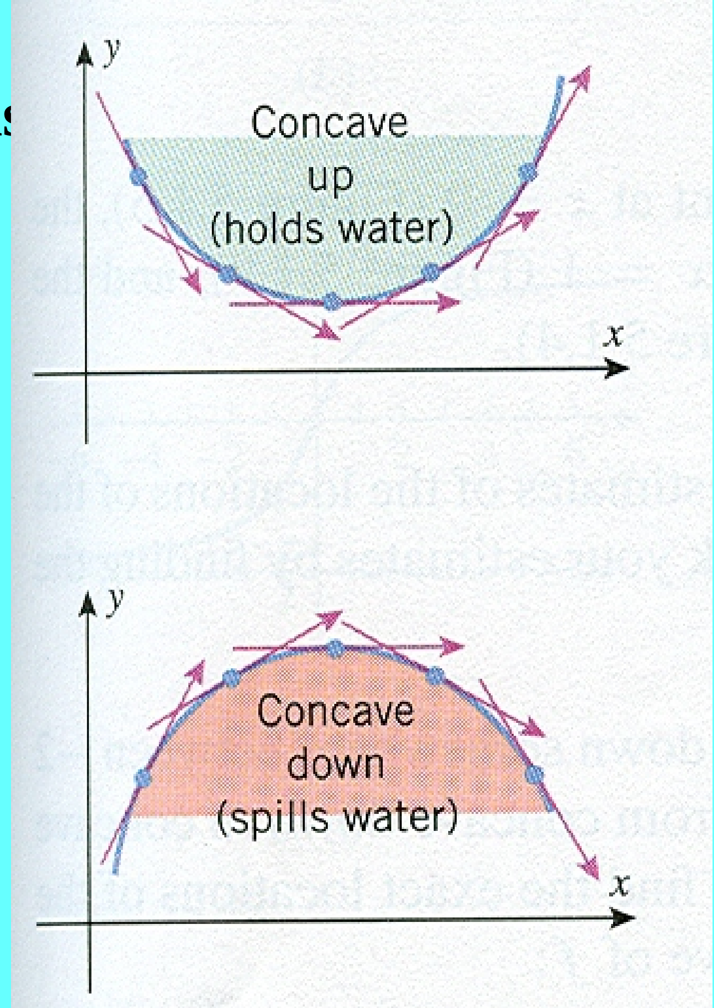
Decreasing: If $x_1 < x_2$, then $f(x_1) > f(x_2)$, f(x) is decreasing.
If $f'(x) < 0$ on the interval, f(x) is decreasing on that interval



Concavity

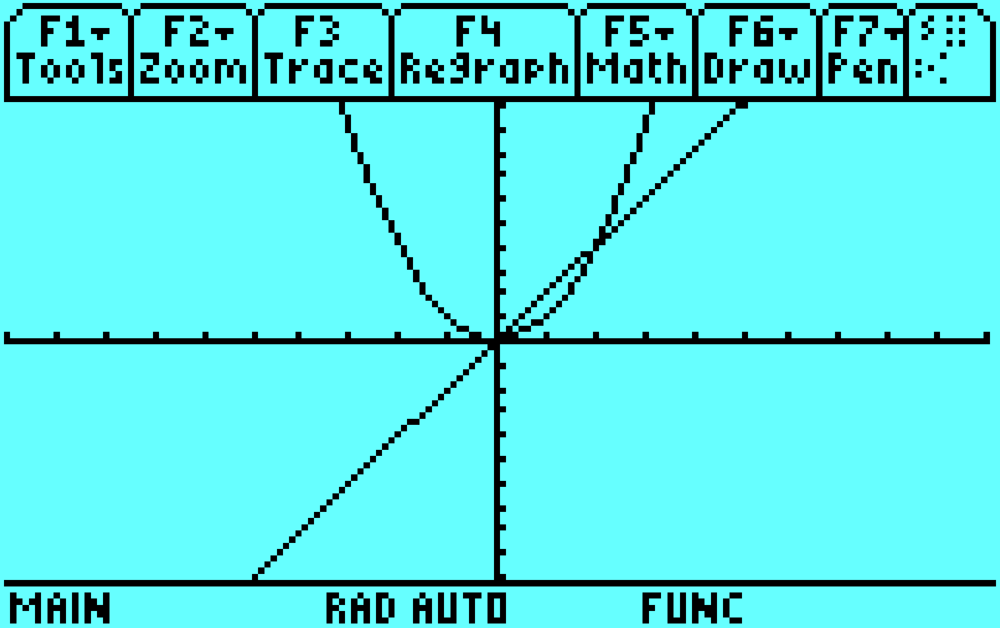
Concave Up: "holds water" or bends upward. Slope of tangent line increasing from left to right.

Concave Down: "spills water" or bends downward: Slope of tangent line decreasing from left to right.



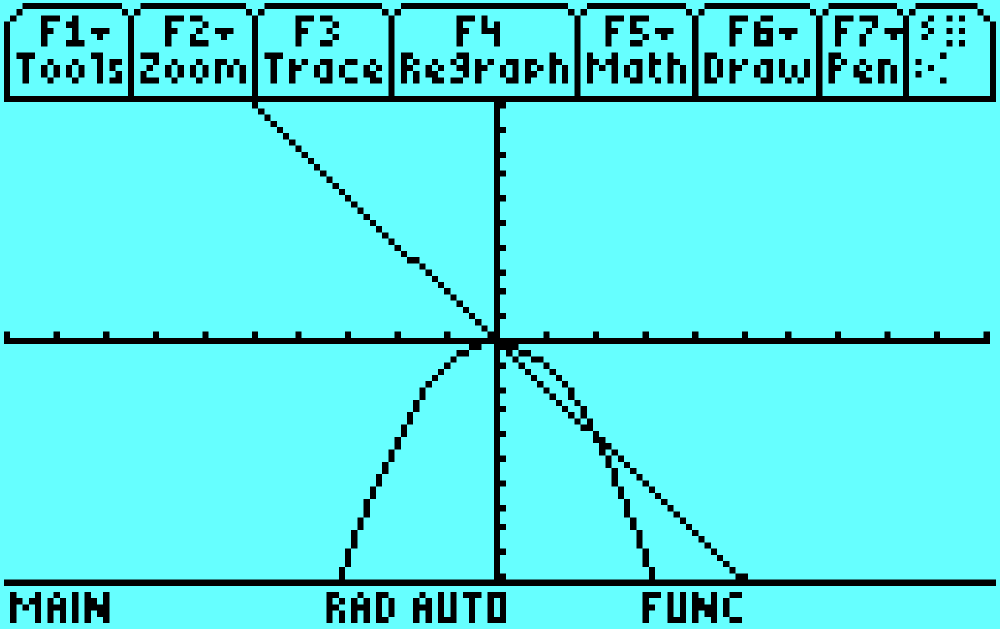
$$y = x^2$$

$$y' = 2x$$



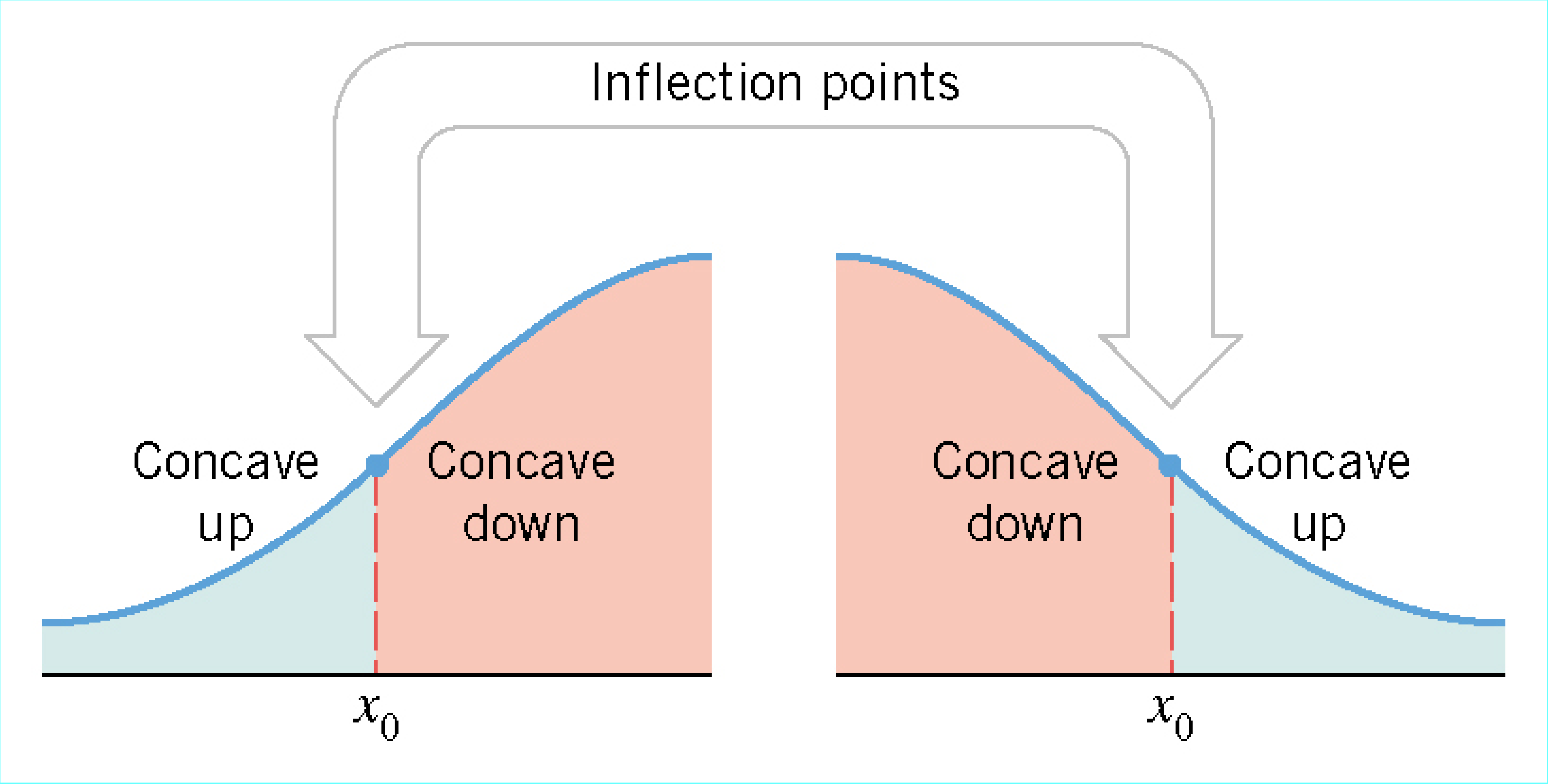
$$y = -x^2$$

$$y' = -2x$$

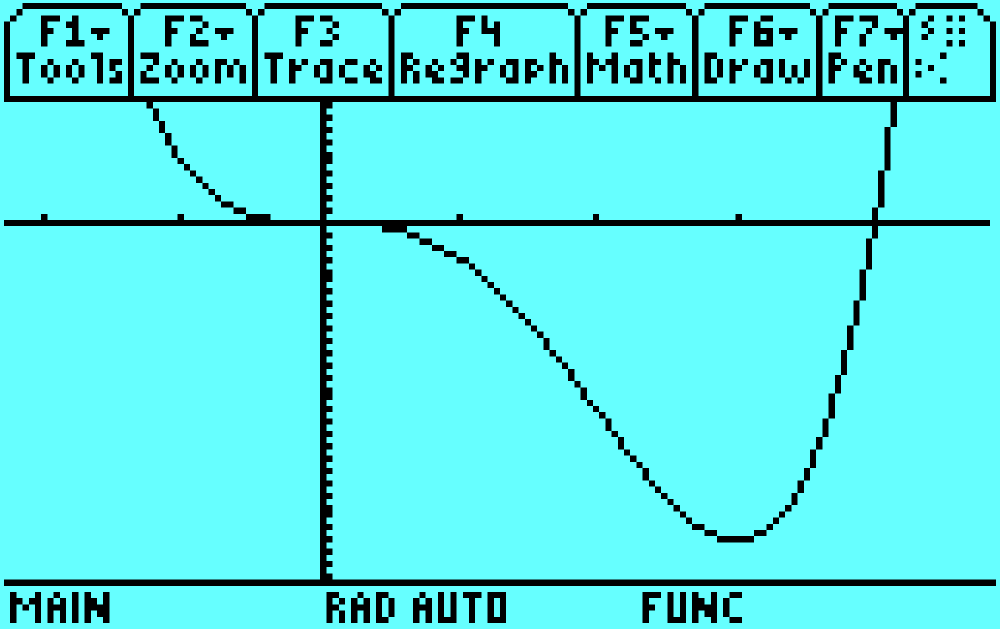


Inflection Point

Point at which concavity switches direction.

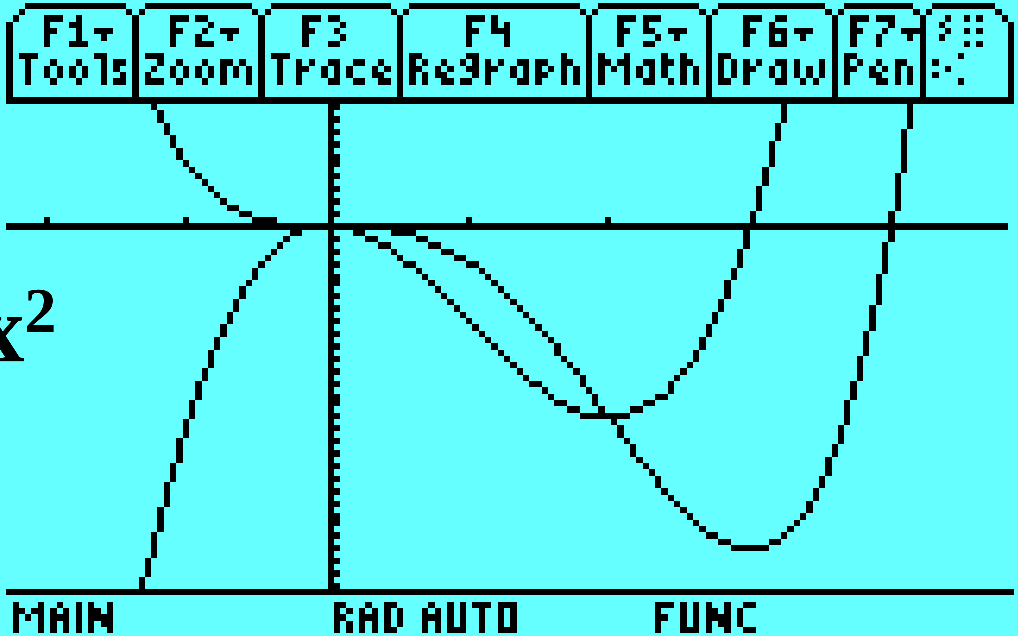


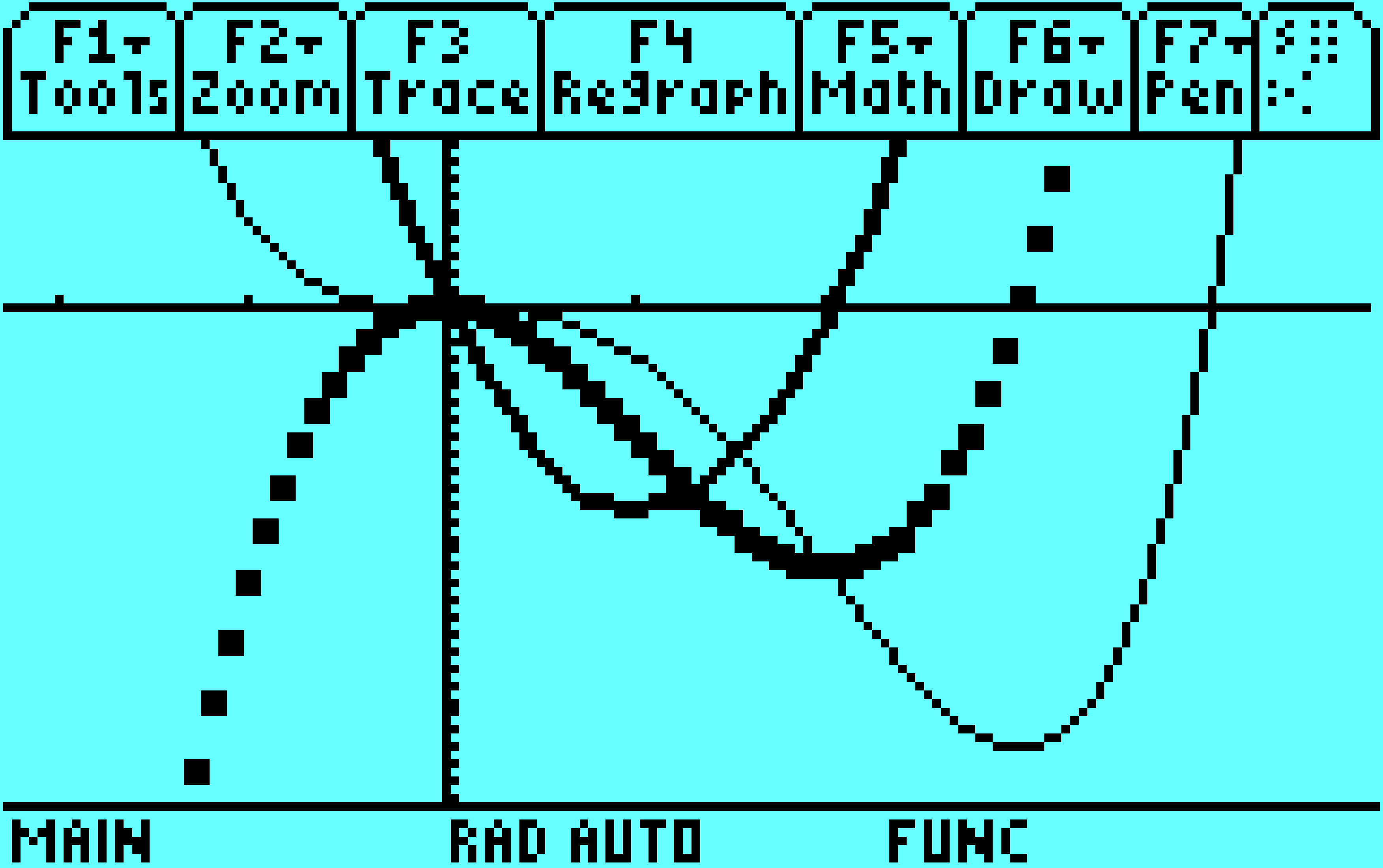
$y = x^4 - 4x^3$



$y = x^4 - 4x^3$

$y' = 4x^3 - 12x^2$





$y = x^4 - 4x^3$ $\quad y' = 4x^3 - 12x^2$ $\quad y'' = 12x^2 - 24x$

TERMINOLOGY

Critical Numbers/Points

Values of x at which f'(x) = 0 or f(x) is undifferentiable.

Relative Extrema

Relative Maximum: f(x) is the larges value in an interval.

Relative Minimum: f(x) is the smallest value in an interval.

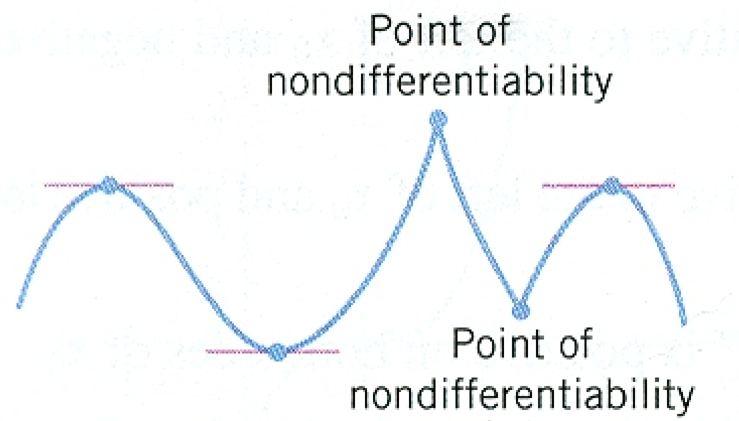


Figure 5.2.3

First Derivative Test

If x_0 is a critical number:

- **If f'(x) > 0 to the immediate left of x_0 and <0 to the immediate right, there is a relative maximum at x_0.**
- **If f'(x) < 0 to the immediate left of x_0 and >0 to the immediate right, there is a relative minimum at x_0.**
- **If f'(x) has the same sign to the left and right of x_0, there is no relative extrema at x_0**

Second Derivative Test

If $f(x)$ is twice differentiable at critical point x_0 :

1. **If $f'(x_0) = 0$ and $f''(x_0) > 0$, there is a relative maximum at x_0**
2. **If $f'(x_0) = 0$ and $f''(x_0) < 0$, there is a relative minimum at x_0**
3. **If $f'(x_0) = 0$ and $f''(x_0) = 0$, the test is inconclusive.**

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$$y = \sin^2 2x$$

$$y = 4\left(\sin^2 x\right)\left(\cos^2 x\right)$$

$$y' = 4\left[\sin^2 x(2\cos x)(-\sin x)+\cos^2 x(2\sin x)(\cos x)\right]$$

$$= 4\left[-2\sin^3 x(\cos x)+2\cos^3 x(\sin x)\right]$$

$$= 4\left[2(\sin x)(\cos x)\right]\left[\cos^2 x-\sin^2 x\right]$$

$$= 4\left[\sin 2x\right]\left[\cos 2x\right]$$

$$= 2\left[2(\sin 2x)(\cos 2x)\right]$$

$$= 2\sin 4x \qquad y''=8\cos 4x$$

$$y'=0 \quad @\ 0,\frac{\pi}{4},\frac{\pi}{2},\frac{3\pi}{4},\pi \qquad y''=0 \quad @\frac{\pi}{8},\frac{3\pi}{8},\frac{5\pi}{8},\frac{7\pi}{8}$$