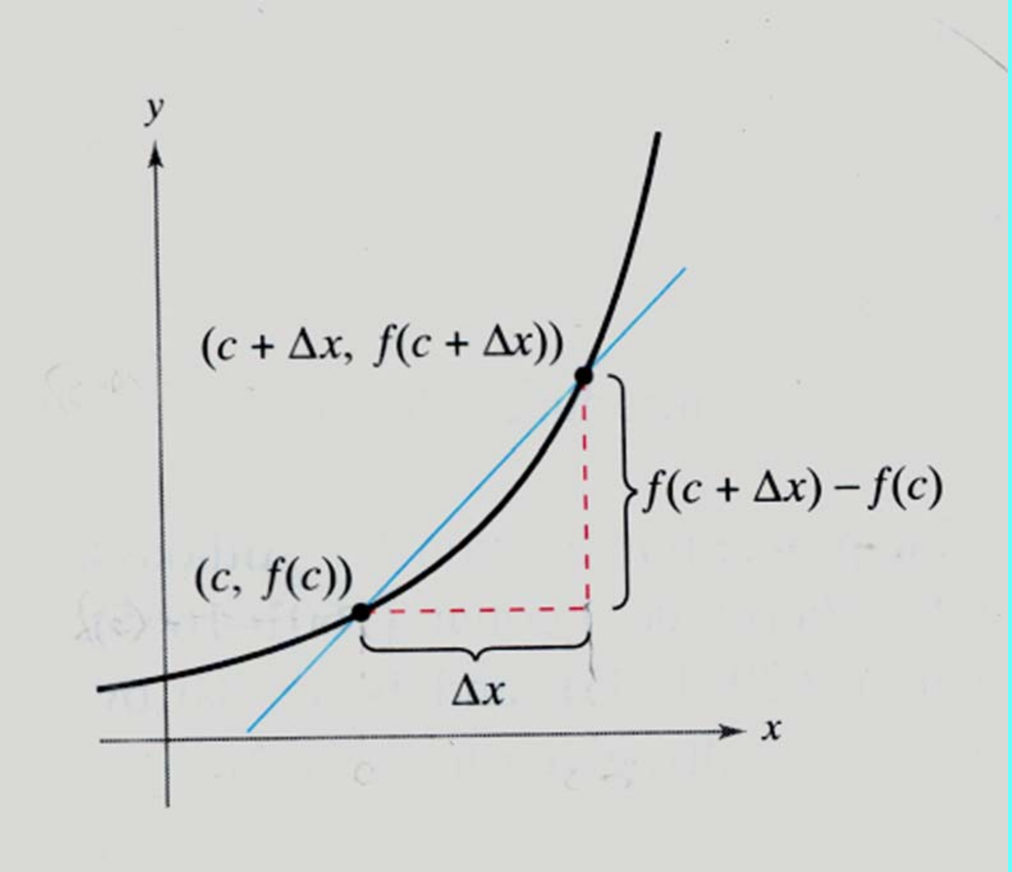
The

Derivative

Slopes

And

Rates of Change



$$m = \frac{f(c+\Delta x) - f(c)}{\Delta x}$$

The

Derivative

Slope of the tangent line to f(x) at any x.

$$\boldsymbol{f'(x) = \underset{\Delta x \to 0}{Limit}\left(\frac{f(x+\Delta x) - f(x)}{\Delta x}\right)}$$

$$f'(x) = \frac{dy}{dx} = y' = \frac{d}{dx}\left[f(x)\right] = D_x\left[y\right]$$

Ex: Find f'(x) for f(x) = x^3 - 2x +1

$$f'(x) = \underset{\Delta x \to 0}{Limit}\left(\frac{f(x+\Delta x) - f(x)}{\Delta x}\right)$$

$$= \underset{\Delta x \to 0}{Limit}\left(\frac{[(x+\Delta x)^3 - 2(x+\Delta x)+1] - [x^3 - 2x + 1]}{\Delta x}\right)$$

$$= \underset{\Delta x \to 0}{Limit}\left(\frac{x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 - 2x - 2\Delta x + 1 - x^3 + 2x - 1}{\Delta x}\right)$$

$$= \underset{\Delta x \to 0}{Limit}\left(\frac{3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 - 2x - 2\Delta x + 1 + 2x - 1}{\Delta x}\right)$$

$$= \underset{\Delta x \to 0}{Limit}\left(\frac{3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 - 2\Delta x}{\Delta x}\right)$$

$$= \underset{\Delta x \to 0}{Limit}\left(3x^2 + 3x\Delta x + \Delta x^2 - 2\right) \quad = 3x^2 - 2$$

BASIC RULES

Of

DIFFERENTIATION

CONSTANT RULE

$$\frac{d}{dx}\left\{c\right\} = \underset{\Delta x \to 0}{Limit} \frac{f\left(x+\Delta x\right)-f\left(x\right)}{\Delta x}$$

$$= \underset{\Delta x \to 0}{Limit} \frac{c-c}{\Delta x} = 0$$

POWER RULE

$$\frac{d}{dx}\left\{x^n\right\} = \underset{\Delta x \to 0}{\text{Limit}}\left\{\frac{(x+\Delta x)^n - x^n}{\Delta x}\right\}$$

$$= \underset{\Delta x \to 0}{\text{Limit}}\frac{x^n + nx^{n-1}(\Delta x) + \dfrac{n(n-1)x^{n-2}}{2}\left(\Delta x^2\right) + ... + (\Delta x)^n - x^n}{\Delta x}$$

$$= \underset{\Delta x \to 0}{\text{Limit}}\left\{nx^{n-1} + \frac{n(n-1)x^{n-2}}{2}(\Delta x) + ... + (\Delta x)^{n-1}\right\}$$

$$= nx^{n-1}$$

CONSTANT MULTIPLE RULE

$$\frac{d}{dx}\left\{cf(x)\right\} = \underset{\Delta x \to 0}{\text{Limit}} \frac{cf\left(x+\Delta x\right)-cf(x)}{\Delta x}$$

$$= \underset{\Delta x \to 0}{\text{Limit}} \quad c\left\{\frac{f\left(x+\Delta x\right)-f(x)}{\Delta x}\right\}$$

$$= c\,\underset{\Delta x \to 0}{\text{Limit}} \left\{\frac{f\left(x+\Delta x\right)-f(x)}{\Delta x}\right\}$$

$$= c\frac{d}{dx}\left(f(x)\right)$$

SUM RULE

$$\frac{d}{dx}\left(f(x)+g(x)\right)=\underset{\Delta x\to 0}{\text{Limit}}\frac{\left\{f\left(x+\Delta x\right)+g\left(x+\Delta x\right)\right\}-\left\{f(x)+g(x)\right\}}{\Delta x}$$

$$=\underset{\Delta x\to 0}{\text{Limit}}\frac{f\left(x+\Delta x\right)+g\left(x+\Delta x\right)-f(x)-g(x)}{\Delta x}$$

$$=\underset{\Delta x\to 0}{\text{Limit}}\left\{\frac{f\left(x+\Delta x\right)-f(x)}{\Delta x}+\frac{+g\left(x+\Delta x\right)-g(x)}{\Delta x}\right\}$$

$$=\underset{\Delta x\to 0}{\text{Limit}}\left\{\frac{f\left(x+\Delta x\right)-f(x)}{\Delta x}\right\}+\underset{\Delta x\to 0}{\text{Limit}}\left\{\frac{g\left(x+\Delta x\right)-g(x)}{\Delta x}\right\}$$

$$=f'(x)+g'(x)$$

DIFFERENCE RULE

$$\frac{d}{dx}\left(f(x)-g(x)\right)=\lim_{\Delta x\to 0}\frac{\left\{f\left(x+\Delta x\right)-g\left(x+\Delta x\right)\right\}-\left\{f(x)-g(x)\right\}}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{f\left(x+\Delta x\right)-g\left(x+\Delta x\right)-f(x)+g(x)}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\left\{\frac{f\left(x+\Delta x\right)-f(x)}{\Delta x}-\frac{g\left(x+\Delta x\right)-g(x)}{\Delta x}\right\}$$

$$=\lim_{\Delta x\to 0}\left\{\frac{f\left(x+\Delta x\right)-f(x)}{\Delta x}\right\}-\lim_{\Delta x\to 0}\left\{\frac{g\left(x+\Delta x\right)-g(x)}{\Delta x}\right\}$$

$$=f'(x)-g'(x)$$

THE PRODUCT RULE

$$\frac{d}{dx}\left\{f(x)g(x)\right\} = \underset{\Delta x \to 0}{Limit}\left\{\frac{f(x+\Delta x)g(x+\Delta x) - f(x)g(x)}{\Delta x}\right\}$$

Adding and subtracting f(x+Δx)g(x):

$$= \underset{\Delta x \to 0}{Limit}\left\{\begin{matrix} \dfrac{\left[f(x+\Delta x)g(x+\Delta x)\right]}{\Delta x} - \dfrac{f(x+\Delta x)g(x)}{\Delta x} + \\ \dfrac{f(x+\Delta x)g(x)}{\Delta x} + \dfrac{\left[-f(x)g(x)\right]}{\Delta x} \end{matrix}\right\}$$

$$= \underset{\Delta x \to 0}{Limit}\left\{f(x+\Delta x)\frac{g(x+\Delta x) - g(x)}{\Delta x} + g(x)\frac{f(x+\Delta x) - f(x)}{\Delta x}\right\}$$

$$= f(x)g'(x) + g(x)f'(x)$$

Product Rule Examples:

1. $$\frac{d}{dx}\left\{\left(x^2-4\right)\left(x^3+3x-5\right)\right\}$$

$$=\left(x^2-4\right)\frac{d}{dx}\left\{x^3+3x-5\right\}+\left(x^3+3x-5\right)\frac{d}{dx}\left\{x^2-4\right\}$$

$$=\left(x^2-4\right)\left(3x^2+3\right)+\left(x^3+3x-5\right)(2x)$$

$$=3x^4-12x^2+3x^2-12+2x^4+6x^2-10x$$

$$=5x^4-3x^2-10x-12$$

The Division Rule

$$\frac{d}{dx}\left\{\frac{f(x)}{g(x)}\right\} = \underset{\Delta \to 0}{Limit}\left\{\frac{\dfrac{f(x+\Delta x)}{g(x+\Delta x)} - \dfrac{f(x)}{g(x)}}{\Delta x}\right\}$$

$$= \underset{\Delta x \to 0}{Limit}\left\{\frac{f(x+\Delta x)g(x) - f(x)g(x+\Delta x)}{\Delta x g(x) g(x+\Delta x)}\right\}$$

Adding and subtracting f(x)g(x):

$$= \underset{\Delta x \to 0}{Limit}\left\{\frac{f(x+\Delta x)g(x) - f(x)g(x) + f(x)g(x) - f(x)g(x+\Delta x)}{\Delta x g(x) g(x+\Delta x)}\right\}$$

Division Rule Continued

$$= \underset{\Delta x \to 0}{Limit}\left\{\frac{f(x+\Delta x)g(x) - f(x)g(x) + f(x)g(x) - f(x)g(x+\Delta x)}{\Delta x g(x)g(x+\Delta x)}\right\}$$

$$= \underset{\Delta x \to 0}{Limit}\left\{\frac{g(x)\left(\frac{f(x+\Delta x) - f(x)}{\Delta x}\right) - f(x)\left(\frac{g(x+\Delta x) - g(x)}{\Delta x}\right)}{g(x)g(x+\Delta x)}\right\}$$

$$= \frac{g(x)f\,'(x) - f(x)g\,'(x)}{g(x)^2}$$

Division Rule Examples:

1. $$\frac{d}{dx}\left\{\frac{x^2-3}{x^3+4}\right\} = \frac{\left(x^3+4\right)(2x)-\left(x^2-3\right)\left(3x^2\right)}{\left(x^3+4\right)^2}$$

$$= \frac{-x^4+9x^2+8x}{\left(x^3+4\right)^2}$$

2. $$\frac{d}{dx}\left\{\frac{\sin x}{\cos x}\right\} = \frac{\cos x(\cos x) - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

Derivatives

of

Trigonometric Functions

DERIVATIVE OF SINE

$$\frac{d}{dx}\left(\sin x\right)=\underset{\Delta x\to 0}{\text{Limit}}\frac{\sin(x+\Delta x)-\sin x}{\Delta x}$$

$$=\underset{\Delta x\to 0}{\text{Limit}}\frac{\sin x\cos\Delta x+\cos x\sin\Delta x-\sin x}{\Delta x}$$

$$=\underset{\Delta x\to 0}{\text{Limit}}\frac{\cos x\sin\Delta x-\sin x(1-\cos\Delta x)}{\Delta x}$$

$$=\underset{\Delta x\to 0}{\text{Limit}}\left\{\frac{\cos x\sin\Delta x}{\Delta x}-\frac{\sin x\left(1-\cos\Delta x\right)}{\Delta x}\right\}$$

$$=\underset{\Delta x\to 0}{\text{Limit}}\left\{\cos x\frac{\sin\Delta x}{\Delta x}-\sin x\frac{\left(1-\cos\Delta x\right)}{\Delta x}\right\}$$

$$=\cos x$$

DERIVATIVE OF COSINE

$$\frac{d}{dx}(\cos x) = \operatorname*{Limit}_{\Delta x \to 0} \frac{\cos(x + \Delta x) - \cos x}{\Delta x}$$

$$= \operatorname*{Limit}_{\Delta x \to 0} \frac{\cos x \cos \Delta x - \sin x \sin \Delta x - \cos x}{\Delta x}$$

$$= \operatorname*{Limit}_{\Delta x \to 0} \left\{ \frac{\cos x(\cos \Delta x - 1)}{\Delta x} - \sin x \frac{\sin \Delta x}{\Delta x} \right\}$$

$$= 0 - \sin x(1) = -\sin x$$

Derivative of Tan x

$$\frac{d}{dx}\{\tan x\} = \frac{d}{dx}\left\{\frac{\sin x}{\cos x}\right\}$$

$$= \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} = \sec^2 x$$

Derivative of sec x

$$\frac{d}{dx}\{\sec x\} = \frac{d}{dx}\left\{\frac{1}{\cos x}\right\}$$

$$= \frac{\cos x(0) - (-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x}\left(\frac{\sin x}{\cos x}\right)$$

$$= \sec x \tan x$$

Similarly

$$\frac{d}{dx}\{\cot x\} = -\csc^2 x$$

$$\frac{d}{dx}\{\csc x\} = -\csc x \cot x$$

Chain Rule

The Chain Rule

If z = g(x) and y = f(z), then y = f(g(x)). If x changes by a small amount Δx then z changes by Δz, then y changes by Δy. The rate change of z with respect to x is:

$$\frac{\Delta z}{\Delta x}$$

The rate of change of y with respect to z is:

$$\frac{\Delta y}{\Delta z}$$

and

$$\frac{\Delta y}{\Delta x} = \frac{\Delta y}{\Delta z}\frac{\Delta z}{\Delta x}$$

Chain Rule Examples:

1. $\frac{d}{dx}\left\{\sqrt{x^2-1}\right\}$ $\quad u = x^2 - 1 \qquad y = \sqrt{u}$

$$\frac{du}{dx} = 2x$$

$$\frac{dy}{du} = \frac{1}{2}(u)^{-\frac{1}{2}} = \frac{1}{2}\left(x^2-1\right)^{-\frac{1}{2}} = \frac{1}{2\sqrt{x^2-1}}$$

$$\frac{dy}{dx} = 2x\left\{\frac{1}{2\sqrt{x^2-1}}\right\} = \frac{x}{\sqrt{x^2-1}}$$

2. $\frac{d}{dx}\{\sin 2x\} = \cos 2x\left\{\frac{d}{dx}2x\right\} = 2\cos 2x$

Ex: Differentiate $(x^3 - 1)^{100}$

$$\frac{d}{dx}\left(x^3 - 1\right)^{100} = 100(x^3 - 1)^{99}\,3x^2 = 300x^2(x^3 - 1)^{99}$$

Ex: find f' (x) if $f(x) = \dfrac{1}{\sqrt[3]{x^2 + x + 1}}$

$$= \left(x^2 + x + 1\right)^{-\frac{1}{3}}$$

$$= -\frac{1}{3}\left(x^2 + x + 1\right)^{-\frac{4}{3}}(2x + 1)$$

Implicit Differentiation

IMPLICIT DIFFERENTIATION

$$x^2+y^2=25$$

$$\frac{d}{dx}\left\{x^2\right\}+\frac{d}{dx}\left\{y^2\right\}=\frac{d}{dx}\left\{25\right\}$$

Using the chain rule:

$$2x+2y\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{-2x}{2y}=-\frac{x}{y}$$

$$x^2+y^2=25$$

$$y^2=25-x^2$$

$$y=\pm\sqrt{25-x^2}$$

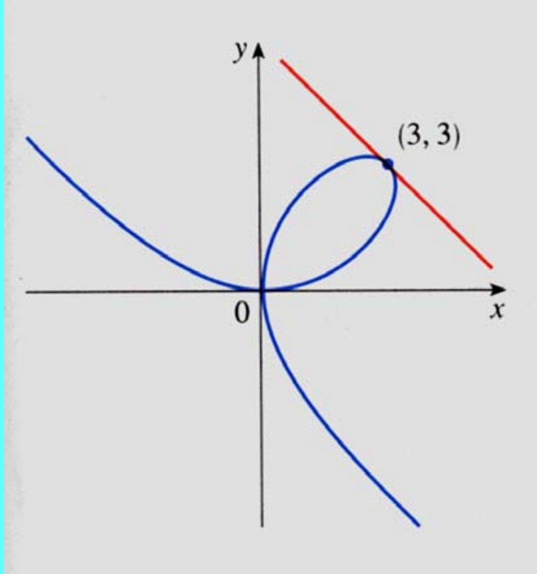
$$\frac{dy}{dx}=\pm\frac{1}{2}\left(25-x^2\right)^{-\frac{1}{2}}\left(-2x\right)$$

$$\frac{dy}{dx}=\pm\frac{\left(x\right)}{\left(25-x^2\right)^{\frac{1}{2}}}=\pm\frac{x}{y}$$

Knowing graph, if x & y same sign slope is negative and if different they are positive indicating -x/y is correct.

Implicit Differentiation Examples:

1. Find the slope of the tangent line to the folium of Decartes at (3,3).



$$x^3+y^3=6xy$$

$$3x^2+3y^2\frac{dy}{dx}=6\left(x\frac{dy}{dx}+y(1)\right)$$

$$3y^2\frac{dy}{dx}-6x\frac{dy}{dx}=6y-3x^2$$

$$\left(3y^2-6x\right)\frac{dy}{dx}=6y-3x^2$$

$$\frac{dy}{dx}=\frac{6y-3x^2}{3y^2-6x}=\frac{2y-x^2}{y^2-2x}$$

$$\frac{dy}{dx}=\frac{2(3)-3^2}{3^2-2(3)}=-1$$

Implicit Differentiation Examples:

2. Find y' if sin(x+y) = y^2cosx

$$\cos(x+y)\left(1+\frac{dy}{dx}\right)=2y\frac{dy}{dx}\cos x+y^2(-\sin x)$$

$$\cos(x+y)+\cos(x+y)\frac{dy}{dx}=2y\frac{dy}{dx}\cos x-y^2\sin x$$

$$\cos(x+y)\frac{dy}{dx}-2y\frac{dy}{dx}\cos x=-y^2\sin x-\cos(x+y)$$

$$\left(\cos(x+y)-2y\cos x\right)\frac{dy}{dx}=-y^2\sin x-\cos(x+y)$$

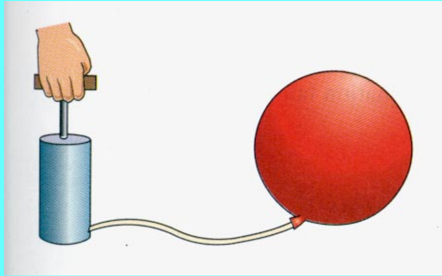
$$\frac{dy}{dx}=\frac{y^2\sin x+\cos(x+y)}{2y\cos x-\cos(x+y)}$$

- Need to add examples of implicit functions

Related Rates

Related Rates

Air is being pumped into a spherical balloon so that its volume increases at a rate of 100 cm^3/sec. How fast is the radius of the balloon increasing when the diameter is 50 cm?



1. **Write an equation relating variables:**

$$V = \frac{4}{3}\pi r^3$$

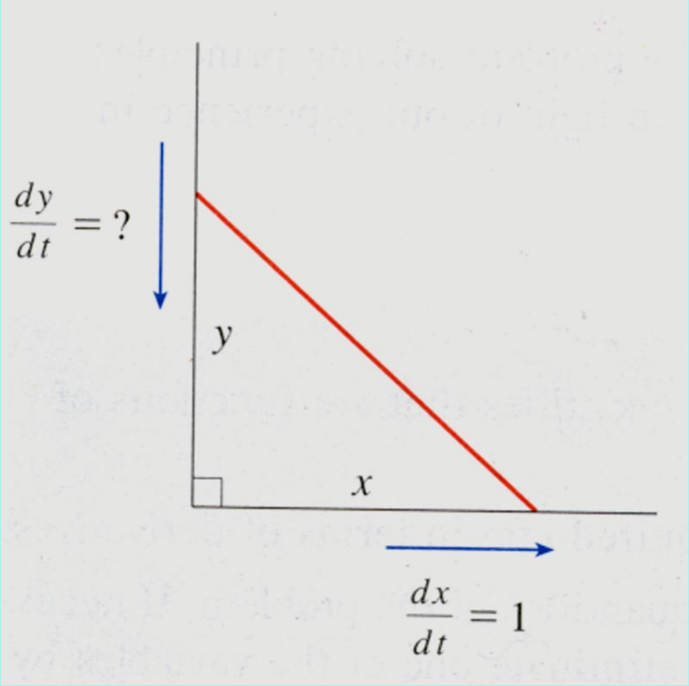
2. **Take the derivative with respect to changing variable:**

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

3. **Substitute information and solve:**

$$100\frac{cm^3}{\sec} = 4\pi 25^2 cm^2 \frac{dr}{dt} \qquad \frac{dr}{dt} = \frac{1}{4\pi 25^2 cm^2} 100\frac{cm^3}{\sec} = \frac{1}{25\pi}\frac{cm}{\sec}$$

2. A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/sec, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall?



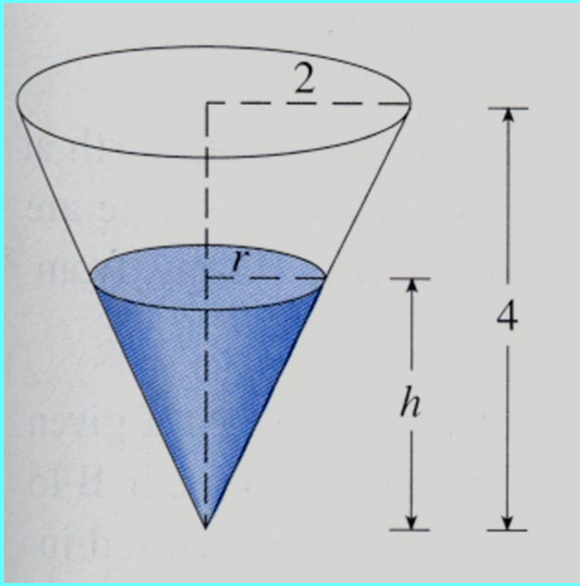
$$x^2 + y^2 = 100$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{2x}{2y}\frac{dx}{dt}$$

$$\frac{dy}{dt} = -\frac{6\text{ft}}{8\text{ft}}\left(1\frac{\text{ft}}{\text{sec}}\right) = -\frac{3}{4}\frac{\text{ft}}{\text{sec}}$$

3. A water tank has a shape of an inverted circular cone with base radius 2 m and a height of 4 m. If water is being pumped into the tank at a rate of 2 m^3/min, find the rate at which the water level is rising when the water 3 m deep.



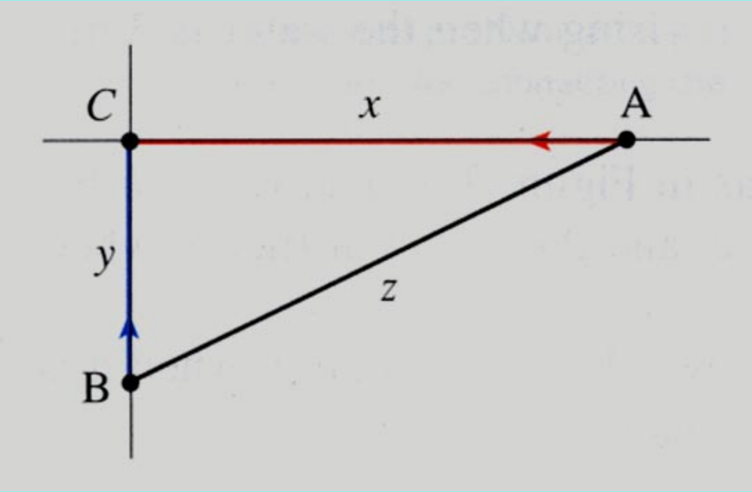
$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{\pi}{12}h^3$$

$$\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{\frac{\pi}{4}h^2}\frac{dV}{dt} = \frac{4}{\pi(3)^2}2 = \frac{8}{9\pi}\frac{m}{min}$$

4. Car A is traveling west at 50 mph and car B is traveling north at 60 mph. Both are headed for the intersection of two roads. At what rate are the cars approaching each other when car A is 0.3 miles and car B is 0.4 miles from the intersection?



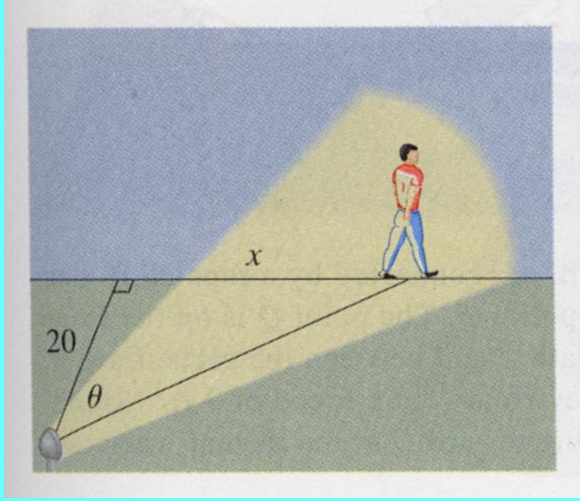
$$z^2 = x^2 + y^2$$

$$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\frac{dz}{dt} = \frac{1}{2z}\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right) = \frac{1}{z}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right)$$

$$\frac{dz}{dt} = \frac{1}{.5}\left(.3(-50) + .4\left(-60\right)\right) = -78\text{mph}$$

5. A man walks along a straight path at a speed of 4 ft/sec. A searchlight is located on the ground 20 ft from the path and is kept focused on the man. At what rate is the search-light rotation when the man is 15 ft from the point on the path closest to the search light?



$$\tan\theta = \frac{x}{20}$$

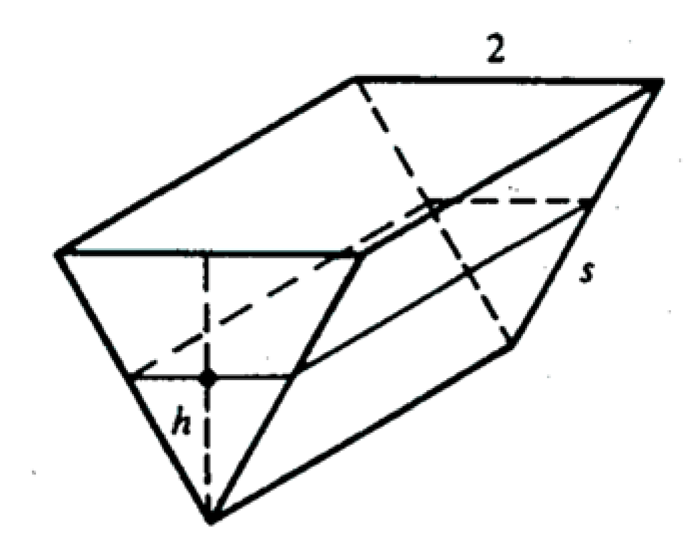
$$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{20}\frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \frac{1}{20\sec^2\theta}\frac{dx}{dt}$$

$$= \frac{1}{20}\cos^2\theta\,(4) = \frac{1}{5}\cos^2\theta$$

$$\frac{d\theta}{dt} = \frac{1}{5}\left(\frac{4}{5}\right)^2 = 0.128 \text{ rad/sec}$$

Ex: A trough is 10 feet long and has a cross section in the shape of an equilateral triangle 2 feet on a side. If water is being pumped in at a rate of 20 ft³/min, how fast is the water level rising when water is 1 foot deep?



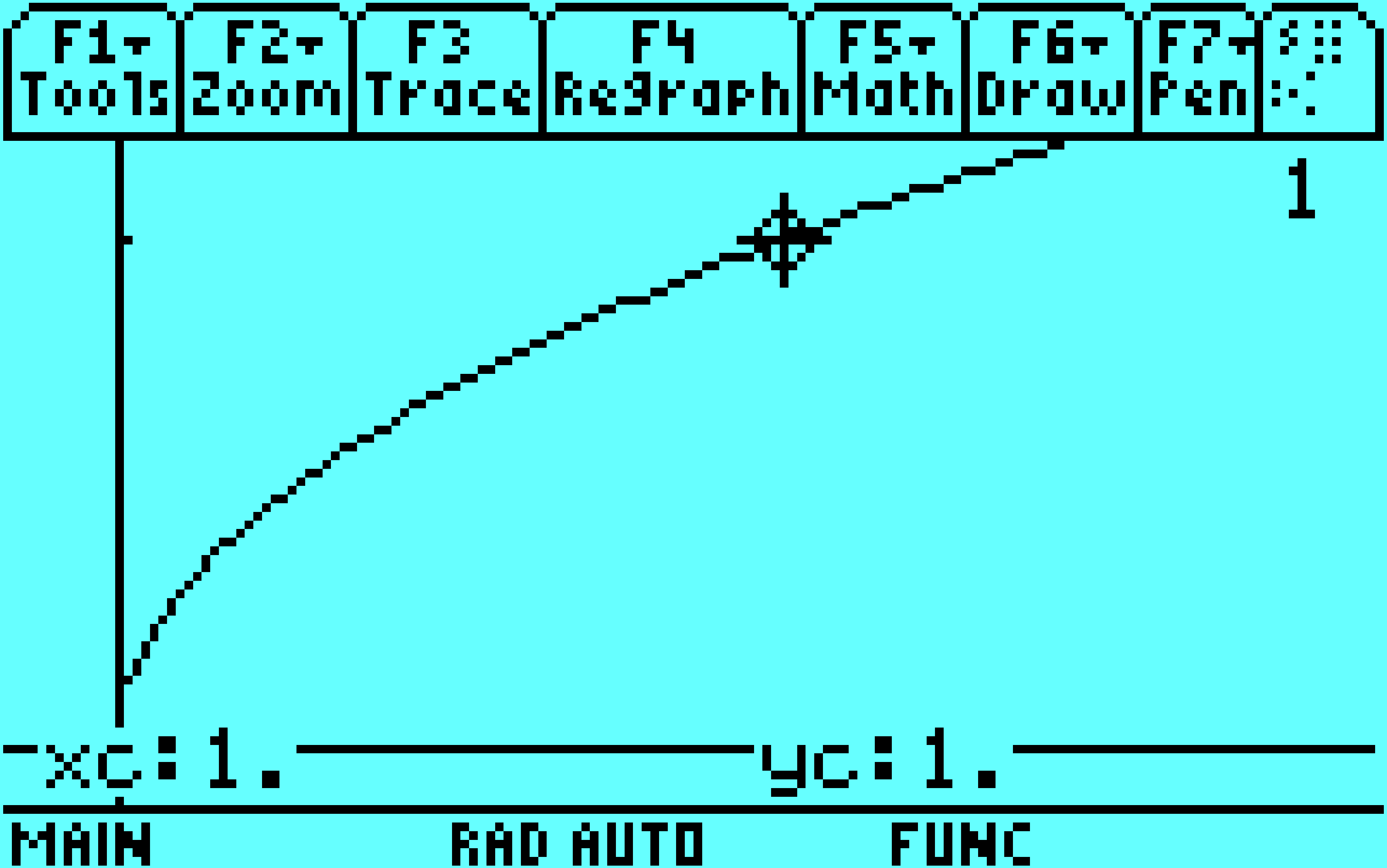
$$h = \frac{s\sqrt{3}}{2} \quad s = \frac{2h}{\sqrt{3}} \quad \text{Area} = \frac{1}{2}sh = \frac{h^2}{\sqrt{3}}$$

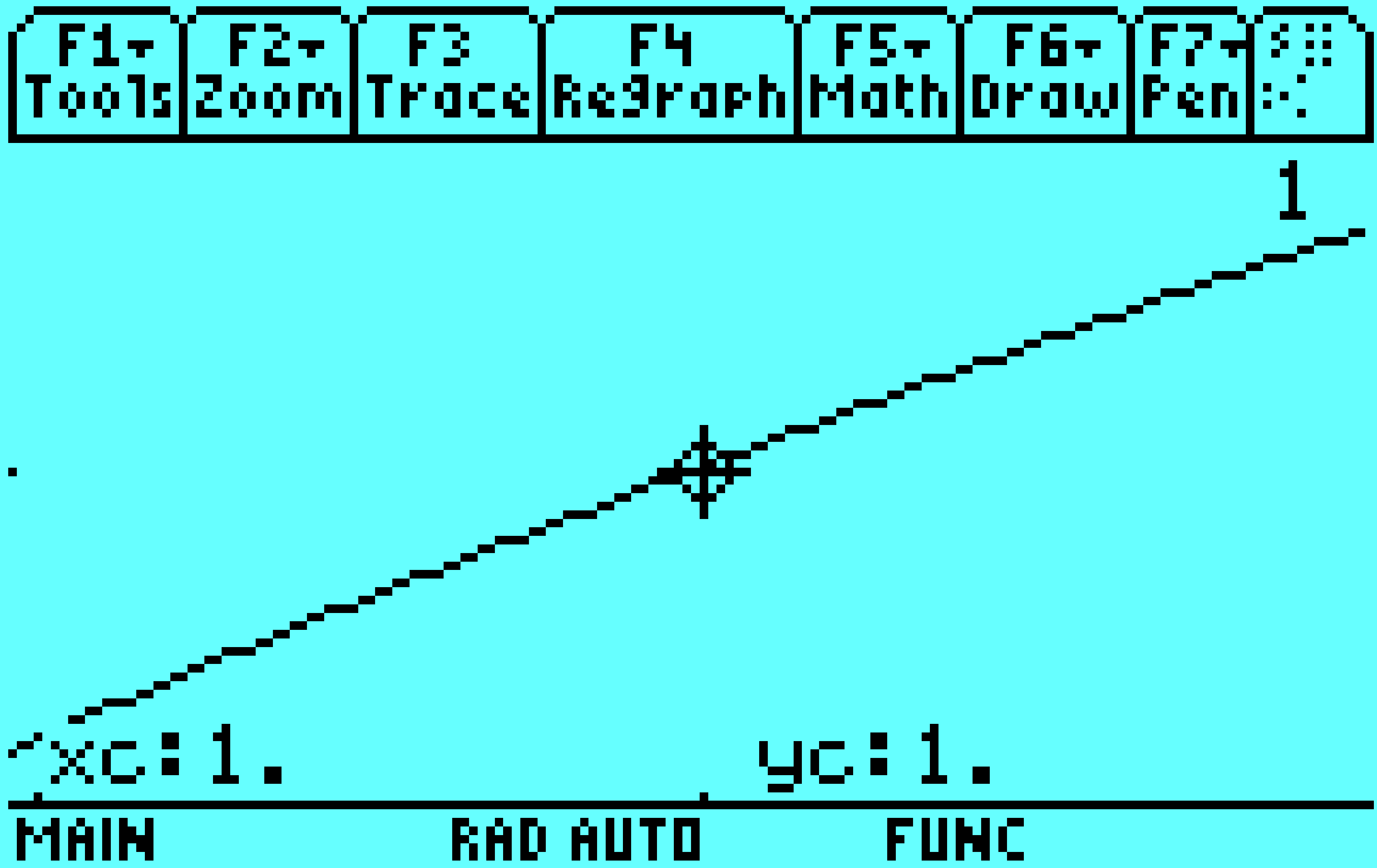
$$V = 10\text{Area} = \frac{10h^2}{\sqrt{3}}$$

$$dV = \frac{20(1)}{\sqrt{3}}dh = 20$$

$$dh = \sqrt{3}$$

Local Linear Approximations


F1 Tools
F2 Zoom
F3 Trace
F4 ReGraph
F5 Math
F6 Draw
F7 Pen
1
xc:1.
yc:1.
MAIN
RAD AUTO
FUNC


F1 Tools
F2 Zoom
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F7 Pen
1
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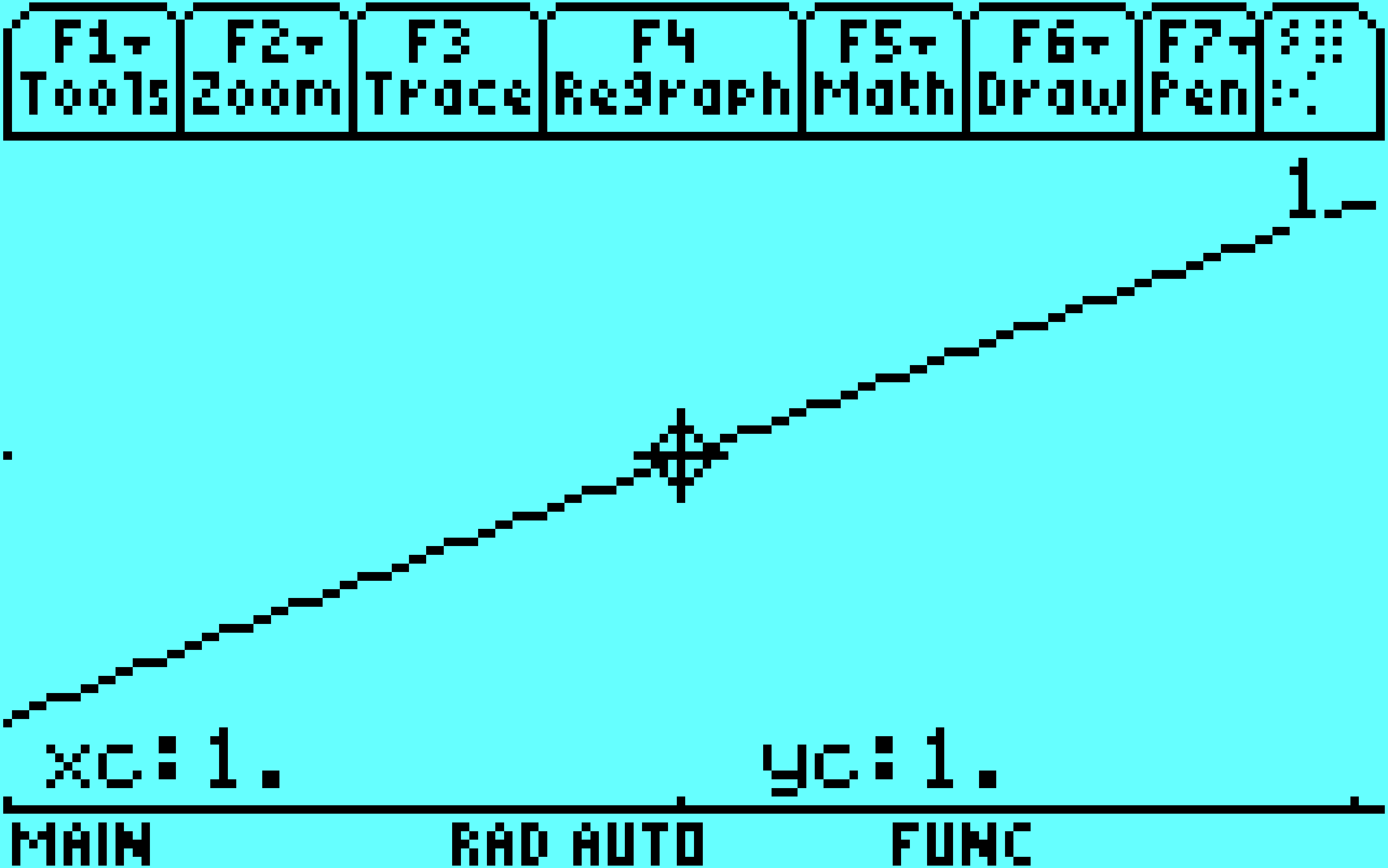
F1 Tools
F2 Zoom
F3 Trace
F4 ReGraph
F5 Math
F6 Draw
F7 Pen
1.
xc:1.
yc:1.
MAIN
RAD AUTO
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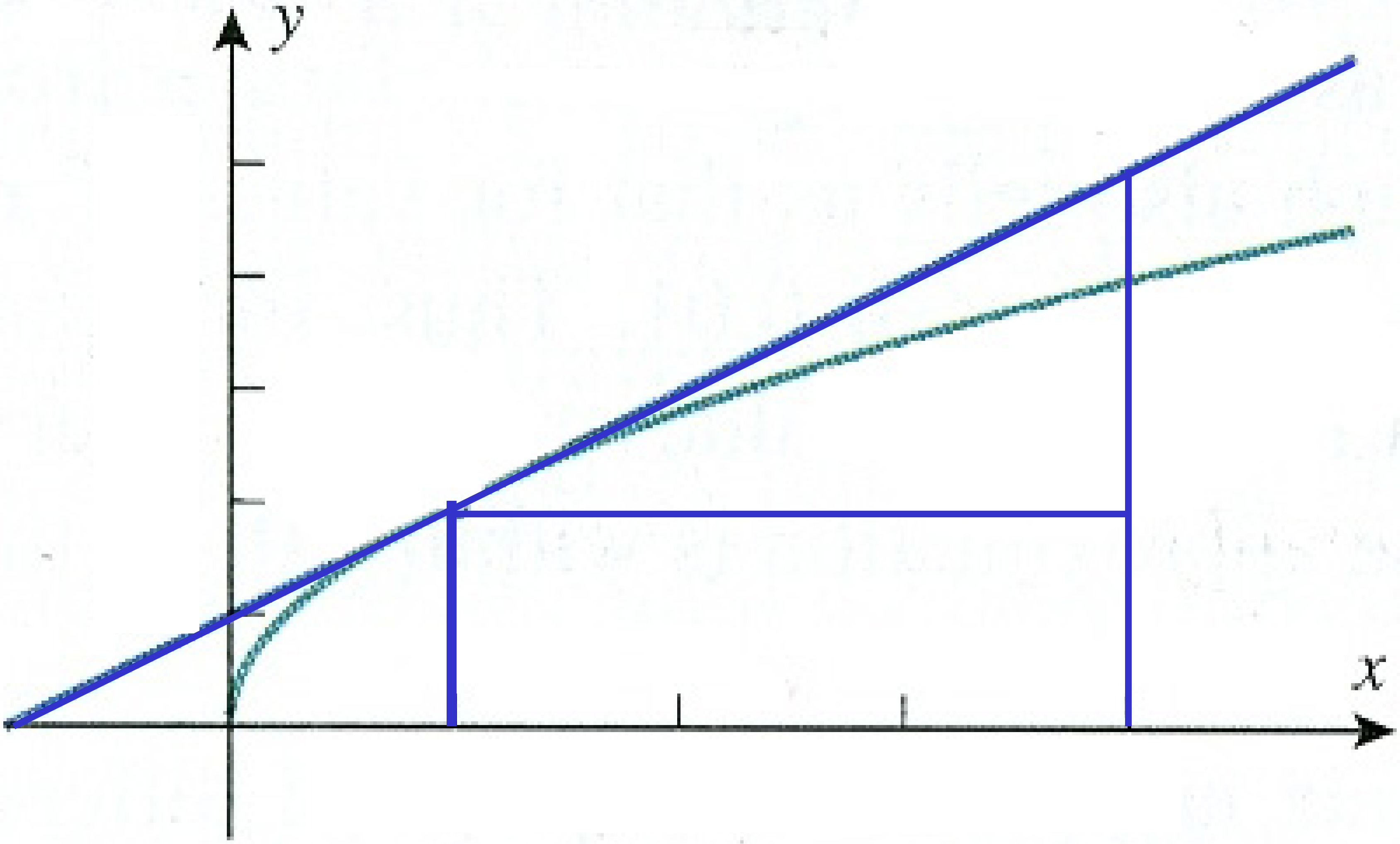
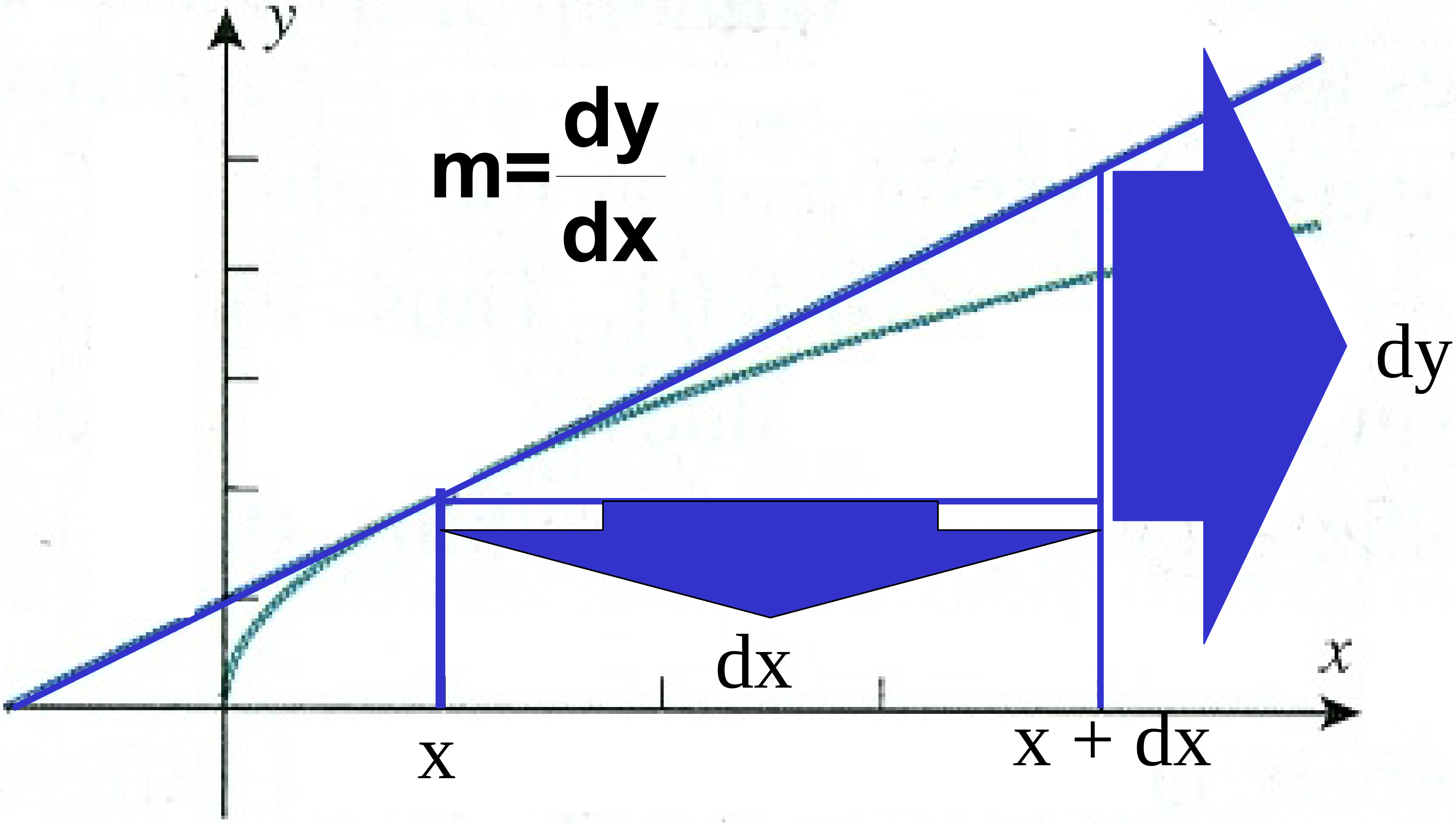
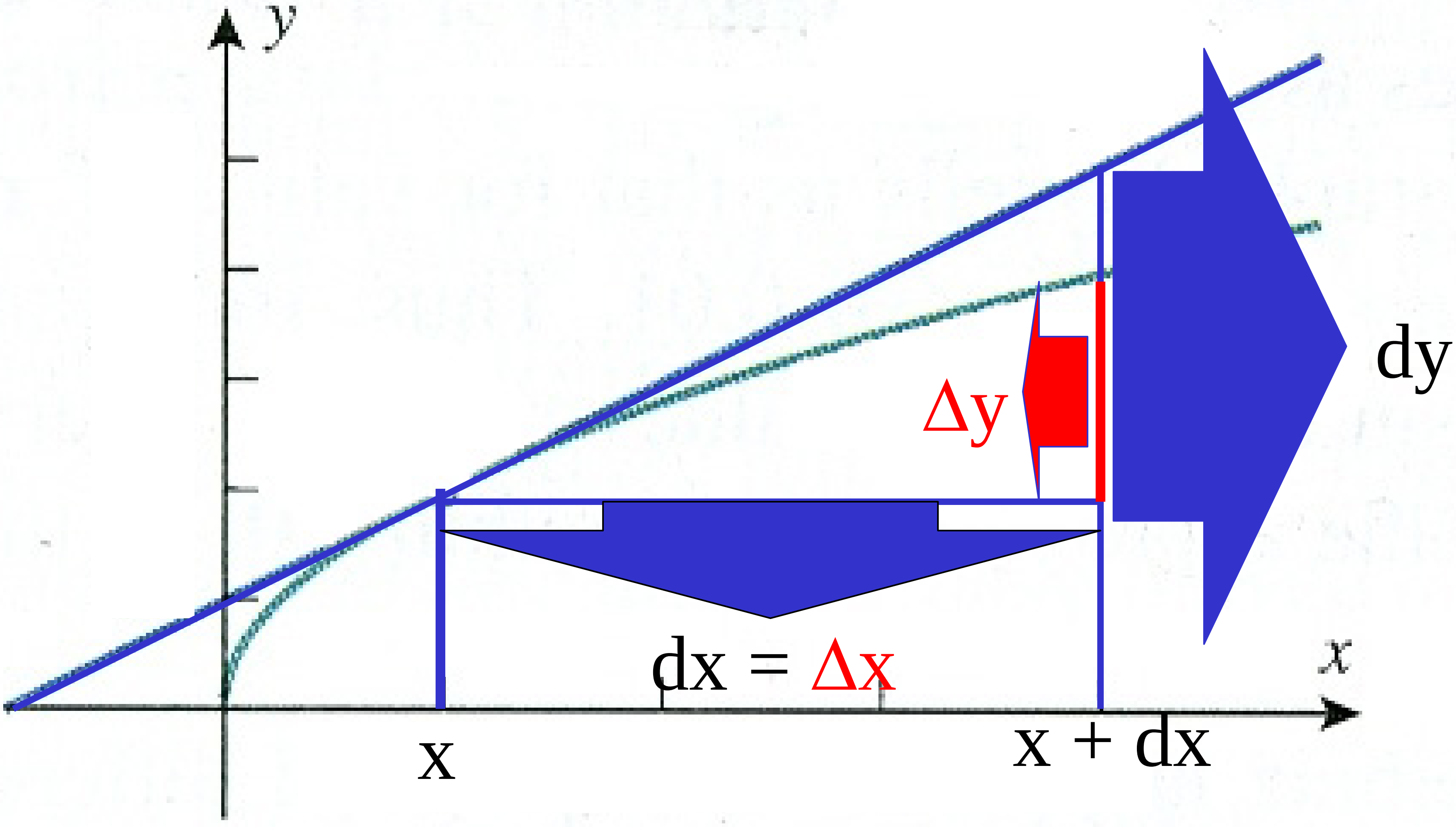

y
x

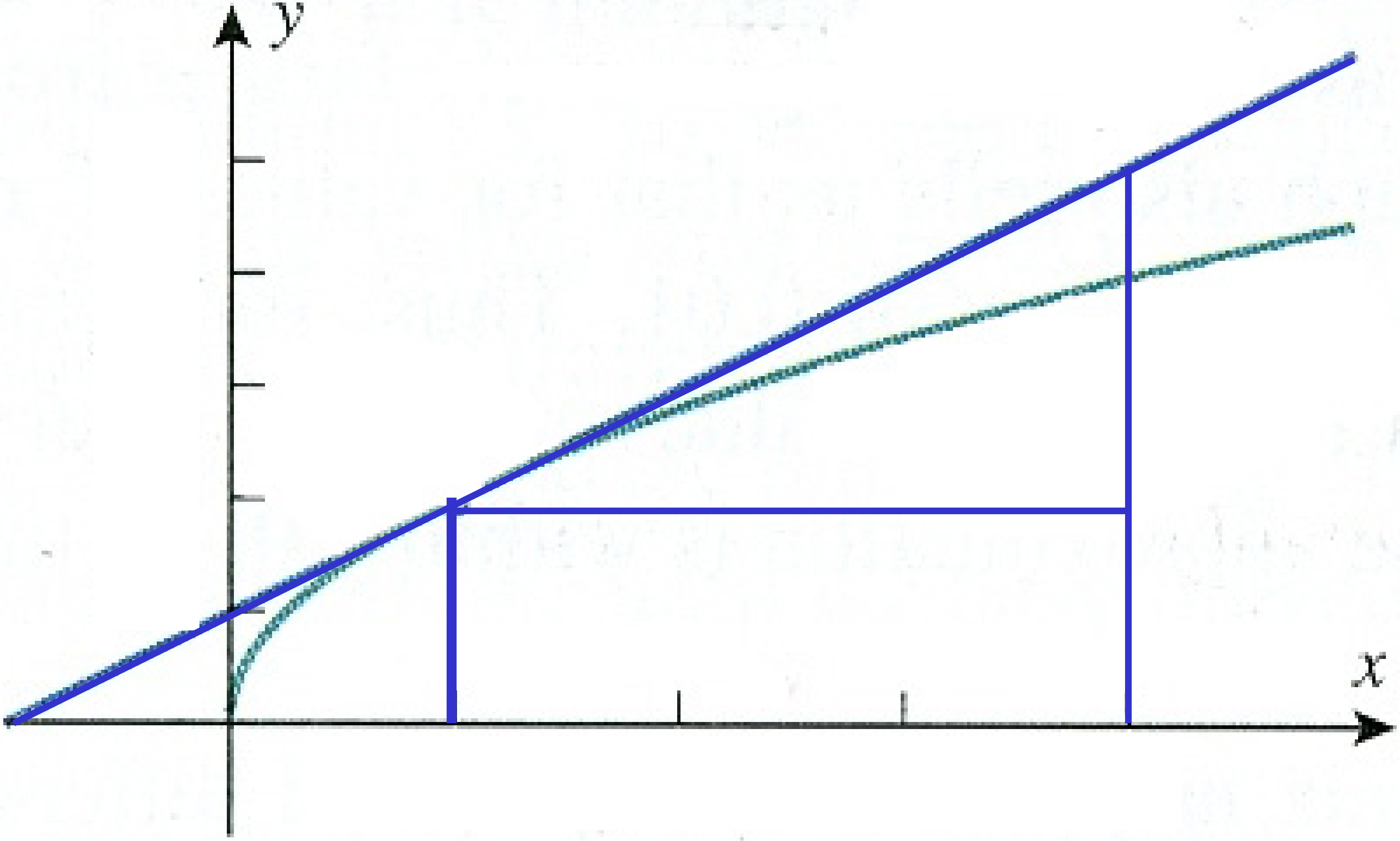


y
m=dy/dx
dy
dx
x
x
x + dx



y
dy
Δy
dx = Δx
x
x
x + dx

How do dy and Δy compare as dx decreases?



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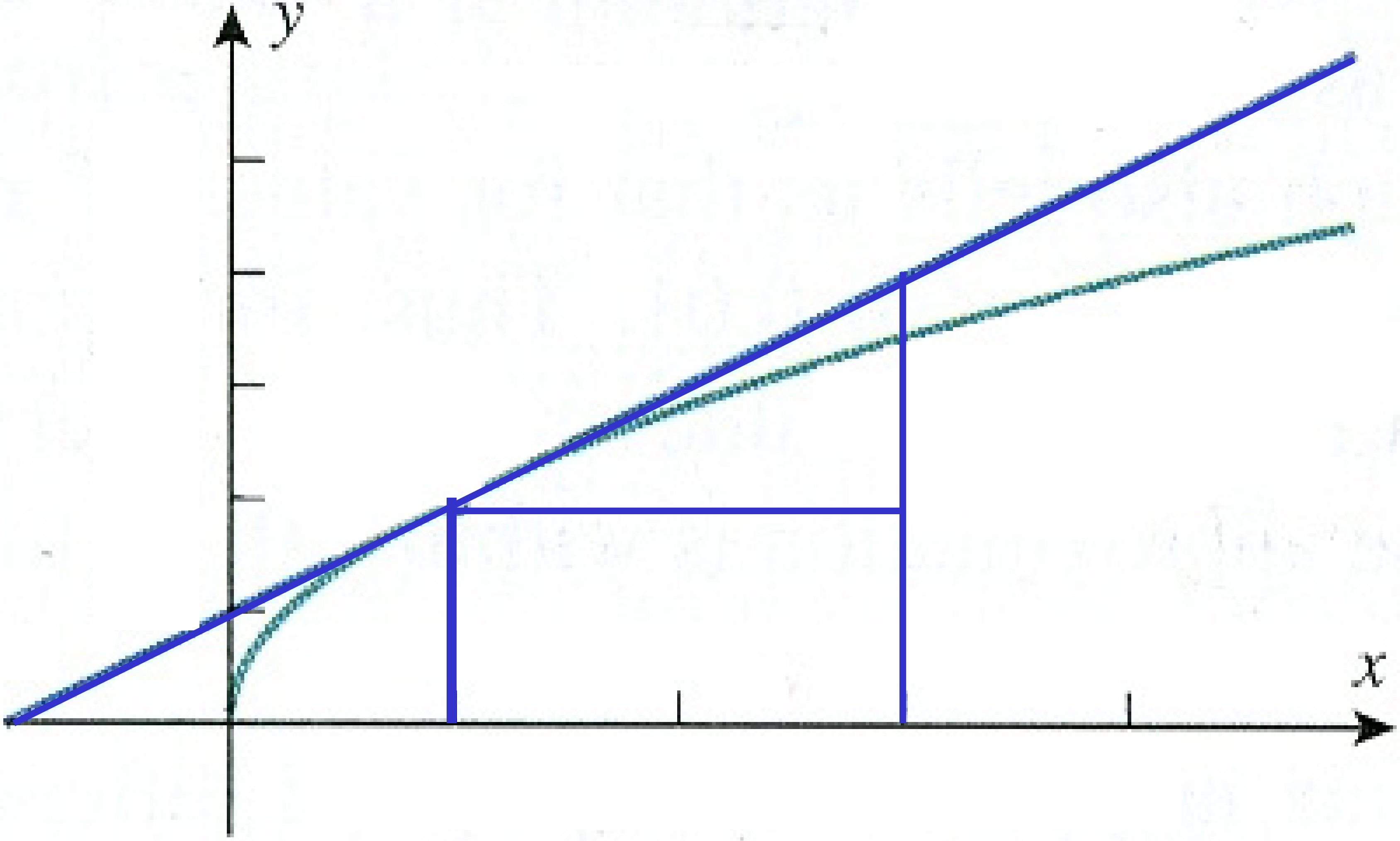
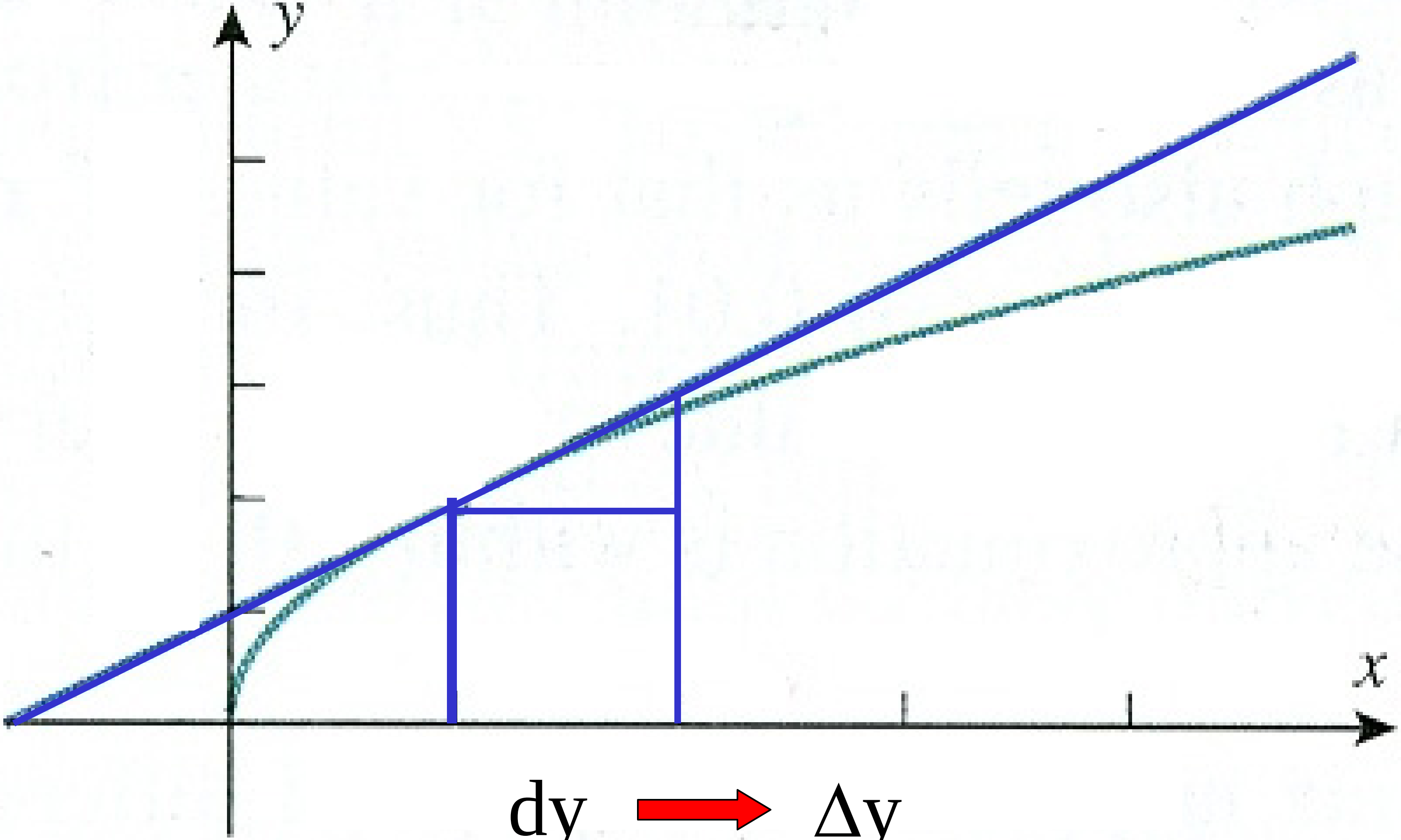


How do dy and Δy compare as dx decreases?



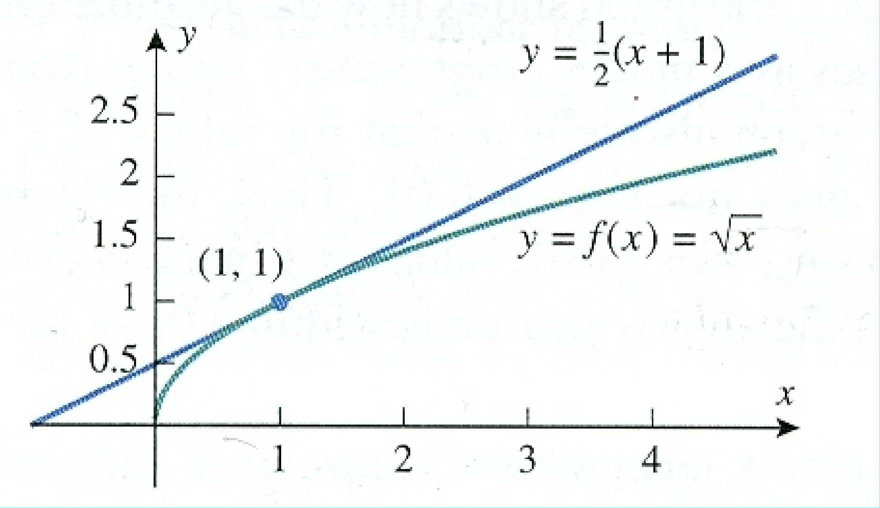
dy → Δy

Using dy to approximate Δy:

As dx approaches zero the equation of the tangent line will be a close approximation for the value of the function.

1. **Find the coordinates of the point of tangency using f(x).**
2. **Find the slope using f ' (x).**
3. **Find the equation of the line using the point-slope form.**
4. **Find the estimate of f(x) using the equation of the line.**

Ex: Find the square root of 1.1



1. Point of tangency is (1, 1)

2. $f(x) = \sqrt{x}$

$$f'(x) = \frac{1}{2\sqrt{x}} \qquad f'(1) = \frac{1}{2}$$

3. $(y-1) = \frac{1}{2}(x-1) \Rightarrow y = \frac{1}{2}x - \frac{1}{2} + 1 = \frac{1}{2}x + \frac{1}{2} = \frac{1}{2}(x+1)$

4. $y = \frac{1}{2}(1.1+1) = \frac{2.1}{2} = 1.05$

Actual: 1.04881

Ex: Find sin 1°

1. Point of tangency is (0, 0)

2. Slope: $f(x) = \sin x$

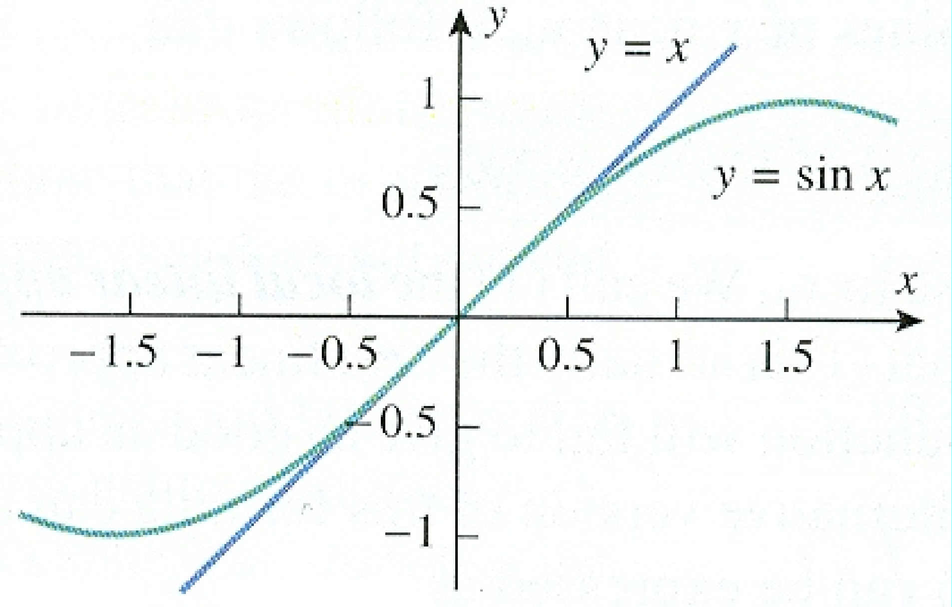
   $f'(x) = \cos x$

   $f'(0) = \cos 0 = 1$

3. Equation: $y = x$

4. Estimate: $y = \dfrac{\pi}{180} = .017453$

   Actual: .017452



Differentials:

dy and dx are considered the “change in y” and “change in x.

Important view is that they are individual quantities, called differentials.

Error Propagation:

Since: $\dfrac{dy}{dx} = f'(x) \quad dy = f'(x)\,dx$

If dx is the error in measuring x, then dy is the error in the calculation of y.

Ex: The radius of a sphere was measured and found to be 21 cm with a possible error in measurement of at most 0.05 cm. What is the maximum error in using this value of the radius to compute the volume of the sphere?

$$V = \frac{4}{3}\pi r^3$$

$$dV = 4\pi r^2 dr$$

$$dV = 4\pi\left(21\right)^2\left(.05\right) = 277$$

Fractional or Relative error is often more informative.
Divide dy by y:

$$\frac{dV}{V} = \frac{4\pi r^2}{\frac{4}{3}\pi r^3} dr = 3\frac{dr}{r}$$

Relative error in Volume is three times the relative error in the radius.

$$\frac{dr}{r} = \frac{.05}{21} = .0024 \qquad \frac{dV}{V} = 3\left(.0024\right) = .007$$

Relative error is often reported as a percentage.

0.24% error in radius

0.7% error in Volume