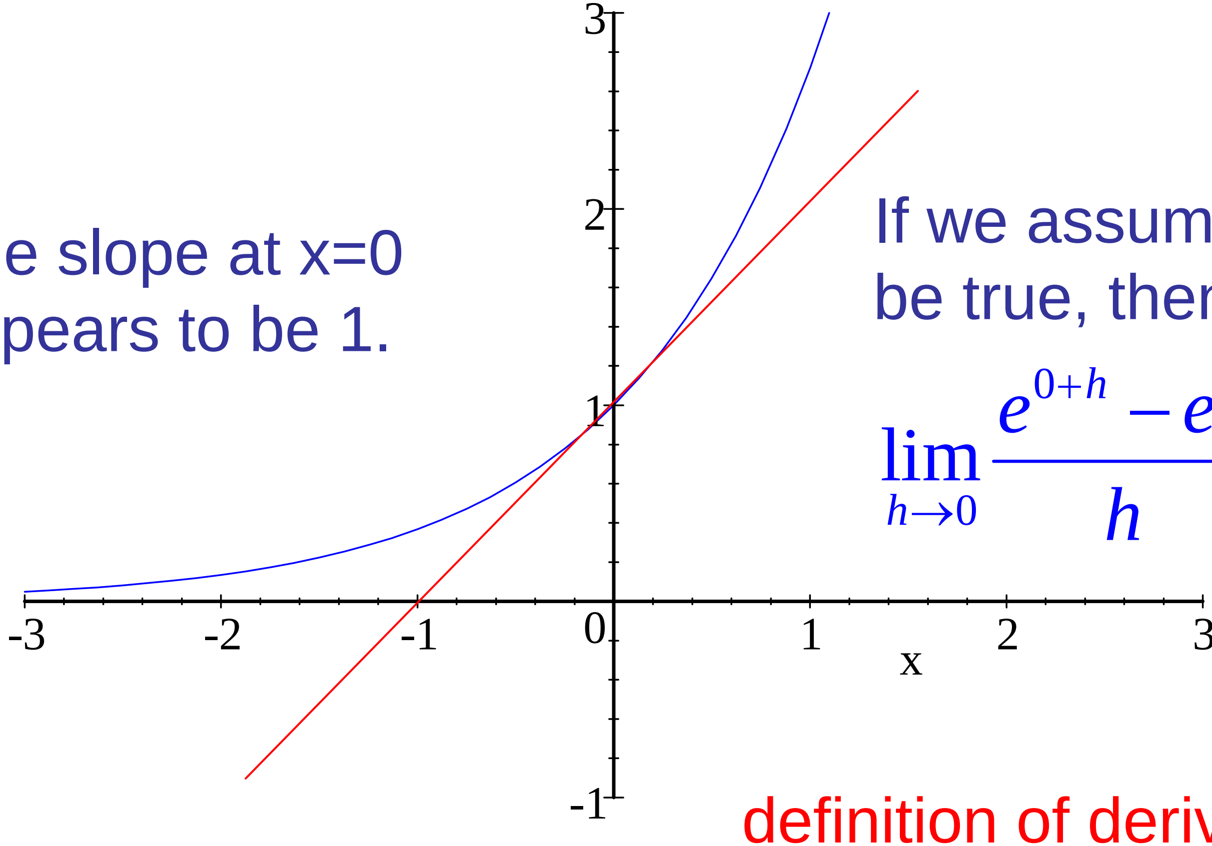
Derivatives of Exponential and Logarithmic Functions

Graph of $y = e^x$

The slope at x=0 appears to be 1.



If we assume this to be true, then:

$$\lim_{h \to 0} \frac{e^{0+h} - e^{0}}{h} = 1$$

definition of derivative

→

Y=(e^x-1)/x

X	Y1
-4E-4	.9998
-3E-4	.99985
-2E-4	.9999
-1E-4	.99995
0	ERROR
1E-4	1.0001
2E-4	1.0001

X=-4E-4

General formula for the derivative of y = e^x

$$\frac{d}{dx}\left(e^{x}\right)=\lim_{h\to 0}\frac{e^{x+h}-e^{x}}{h}$$

$$=\lim_{h\to 0}\frac{e^{x}\cdot e^{h}-e^{x}}{h}$$

$$=\lim_{h\to 0}\left(e^{x}\cdot\frac{e^{h}-1}{h}\right)$$

$$=e^{x}\cdot\lim_{h\to 0}\left(\frac{e^{h}-1}{h}\right) \qquad =e^{x}\cdot 1 \qquad =e^{x}$$

→

$$\frac{d}{dx}\left(e^{x}\right)=e^{x}$$

e^x is its own derivative!

If we incorporate the chain rule:

$$\boxed{\frac{d}{dx}e^u = e^u \frac{du}{dx}}$$

We can now use this formula to find the derivative of a^x

→

Derivative of a^x

$$\frac{d}{dx}\left(a^x\right)$$

$$\frac{d}{dx}\left(e^{\ln a^x}\right)$$ (e^x and $\ln x$ are inverse functions.)

$$\frac{d}{dx}\left(e^{x\ln a}\right)$$

$$e^{x\ln a}\cdot\frac{d}{dx}\left(x\ln a\right)$$ (chain rule)

→

$$\frac{d}{dx}\left(a^x\right)$$

$$\frac{d}{dx}\left(e^{\ln a^x}\right)$$

$$\frac{d}{dx}\left(e^{x\ln a}\right)$$

$$e^{x\ln a}\cdot\frac{d}{dx}\left(x\ln a\right)$$

$$e^{x\ln a}\cdot\ln a$$

$$a^x\cdot\ln a$$

Incorporating the chain rule:

$$\boxed{\frac{d}{dx}\left(a^u\right)=a^u\ln a\frac{du}{dx}}$$

So far today we have:

$$\frac{d}{dx}e^u = e^u \frac{du}{dx}$$

$$\frac{d}{dx}\left(a^u\right) = a^u \ln a \frac{du}{dx}$$

Now it is relatively easy to find the derivative of $\ln x$.

$\rightarrow$

$$y = \ln x$$

$$\frac{dy}{dx} = \frac{1}{e^y}$$

$$e^y = x$$

$$\frac{d}{dx}\left(e^y\right) = \frac{d}{dx}(x)$$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$e^y \frac{dy}{dx} = 1$$

$$\boxed{\frac{d}{dx}\ln u = \frac{1}{u}\frac{du}{dx}}$$

→

To find the derivative of a common log function, you could just use the change of base rule for logs:

$$\frac{d}{dx}\log x \quad = \frac{d}{dx}\frac{\ln x}{\ln 10} \quad = \frac{1}{\ln 10}\frac{d}{dx}\ln x \quad = \frac{1}{\ln 10}\cdot\frac{1}{x}$$

The formula for the derivative of a log of any base other than *e* is:

$$\boxed{\frac{d}{dx}\log_a u = \frac{1}{u\ln a}\frac{du}{dx}}$$

→

$$\frac{d}{dx}e^u = e^u \frac{du}{dx}$$

$$\frac{d}{dx}\left(a^u\right) = a^u \ln a \frac{du}{dx}$$

$$\frac{d}{dx}\log_a u = \frac{1}{u \ln a}\frac{du}{dx}$$