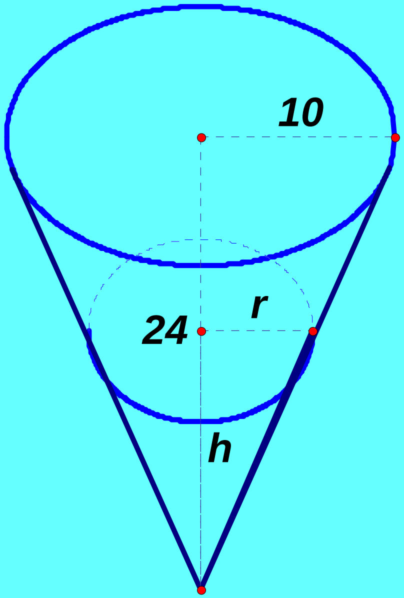


Pbm 25: A conical water tank with vertex down has a radius of 10 ft at the top and is 24 ft high. If water flows into the tank at a rate of 20 ft^3 per minute, how fast is the depth of the water increasing when the water is 16 ft deep?

$$\frac{r}{h}=\frac{10}{24}=\frac{5}{12} \qquad h = \frac{5r}{12}$$

$$V=\frac{\pi}{3}r^2h$$

$$V = \frac{\pi}{3}\left(\frac{5h}{12}\right)^2 h = \frac{25\pi}{432}h^3 \frac{dh}{dt}$$

$$\frac{dV}{dt} = \frac{25\pi}{144}h^2 \frac{dh}{dt}$$

$$20 = \frac{25\pi}{144}16^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{20\left(144\right)}{25\left(16^2\right)\pi} = \frac{9}{20\pi}$$

Pbm 28: Wheat is poured through a chute at the rate of 10 ft^3/min, and falls in a conical pile whose bottom radius is always half of the altitude. How fast will the circumference of the base increasing, when the pile is 8 ft high?

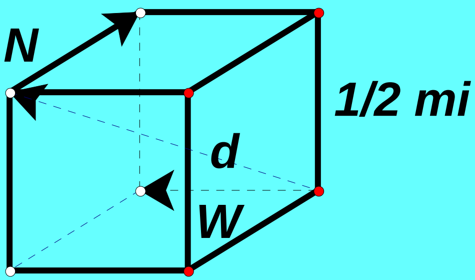
$$V = \frac{1}{3}\pi r^2 h \quad C = 2\pi r = 2\pi\left(\frac{h}{2}\right) = \pi h \quad r = \frac{C}{2\pi} \quad h = \frac{C}{\pi}$$

$$V = \frac{1}{3}\pi\left(\frac{C}{2\pi}\right)^2\left(\frac{C}{\pi}\right) = \frac{C^3}{12\pi^2}$$

$$\frac{dV}{dt} = \frac{C^2}{4\pi^2}\frac{dC}{dt}$$

$$10 = \frac{(8\pi)^2}{4\pi^2}\frac{dC}{dt}$$

$$\frac{dC}{dt} = \frac{10}{16} = \frac{5}{8}$$



Pbm 36: A police helicopter is flying due north at 10 mi/h and at a constant altitude of ½ mi. Below, a car is traveling west on a highway at 75 mi/h. At the moment the helicopter crosses over the highway the car is 2 mi east of the helicopter. (a) How fast is the distance between the car and the helicopter changing at that moment? (b) Is the distance between the car and the helicopter increasing or decreasing?

$$d^2 = N^2 + W^2 + \left(\frac{1}{2}\right)^2$$

$$2d\frac{dd}{dt} = 2N\frac{dN}{dt} + 2W\frac{dW}{dt}$$

$$\frac{dd}{dt} = \frac{N}{d}\frac{dN}{dt} + \frac{W}{d}\frac{dW}{dt}$$

$$\frac{dd}{dt} = \frac{0}{\frac{\sqrt{17}}{2}}100 + \frac{2}{\frac{\sqrt{17}}{2}}(-75) = \frac{4}{\sqrt{17}}(-75) = \frac{-300}{\sqrt{17}}$$

Pbm 37: A particle is moving along the curve whose equation is given below. Assume that the x-coordinate is increasing at the rate of 6 units/sec when the particle is at the point (1, 2). (a) At what rate is the y- coordinate of the point changing at that instant? (b) Is the particle rising or falling at that instant?

$$\frac{xy^3}{1+Y^2}=\frac{8}{5}$$

$$5xy^3=8\left(1+Y^2\right)$$

$$5x\left(3y^2\frac{dy}{dt}\right)+5Y^3\frac{dx}{dt}=16Y\frac{dy}{dt}$$

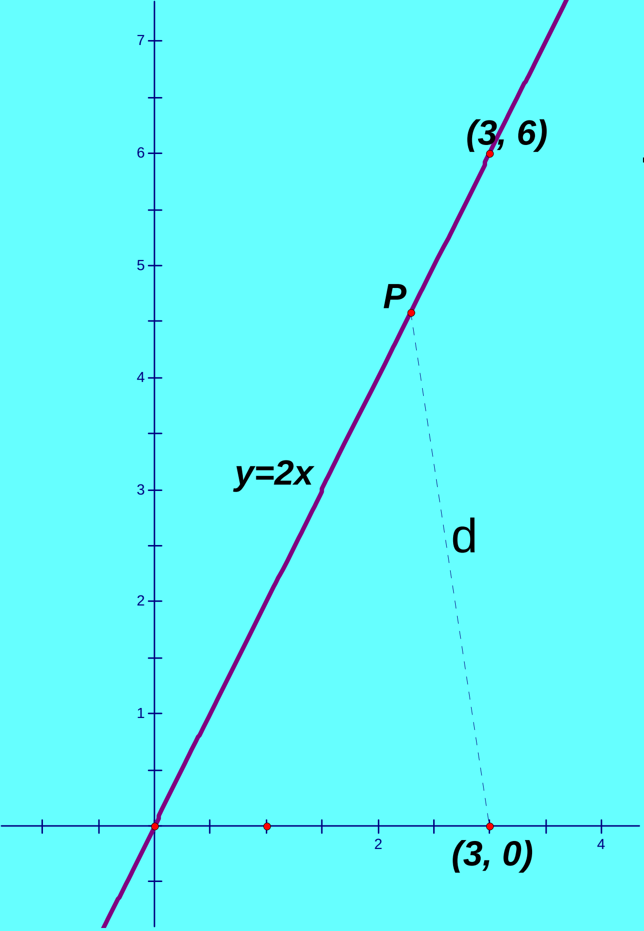
$$15xy^2\frac{dy}{dt}-16Y\frac{dy}{dt}=-5Y^3\frac{dx}{dt}$$

$$\left(15xy^2-16Y\right)\frac{dy}{dt}=-5Y^3\frac{dx}{dt}$$

$$\frac{dy}{dt}=\frac{-5Y^3\frac{dx}{dt}}{\left(15xy^2-16Y\right)}$$

$$\frac{dy}{dt}=\frac{-5Y^3\frac{dx}{dt}}{\left(15xy^2-16Y\right)}\Bigg|_{1,2}=\frac{-5(8)(6)}{15(1)(4)-16(2)}=-\frac{60}{7}$$

Falling

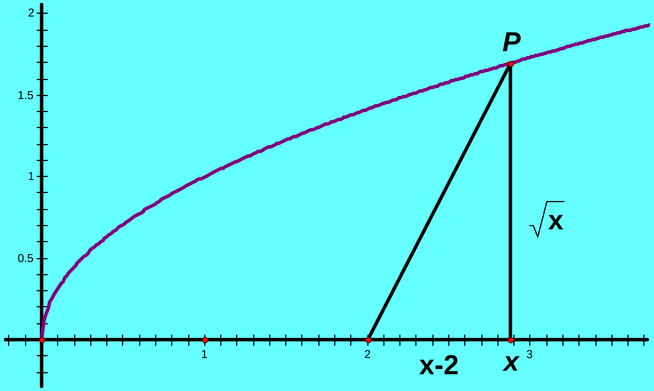


PBM 39: A point P is moving along the line whose equation is y = 2x. How fast is the distance between P and the point (3, 0) changing at the instant when P is at (3, 6) if x is decreasing at the rate of 2 units per second at that instant?

$$d = \sqrt{(x-3)^2+(2x-0)^2} = \sqrt{5x^2-6x+9}$$

$$\frac{dd}{dt}=\frac{1}{2}\left(5x^2-6x+9\right)^{-\frac{1}{2}}(10x-6)\frac{dx}{dt}=\frac{5x-3}{\sqrt{5x^2-6x+9}}\frac{dx}{dt}$$

$$\left.\frac{dd}{dt}\right|_{x=3} = \frac{5(3)-3}{\sqrt{5(3)^2-6(3)+9}}\left(-2\right) = -2$$



Pbm 40: A point P is moving along the curve whose equation is y = sqrt x. Suppose x is increasing at the rate of 4 units per second when x = 3. (a) How fast is the distance between P and the point (2, 0) changing at that instant? (b) How fast is the angle of inclination of the line segment from P to (2, 0) changing at this instant.

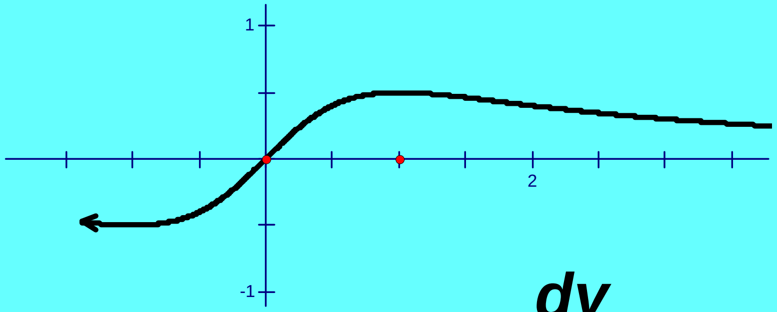
(a) $$d = \sqrt{(x-2)^2 + (\sqrt{x})^2}$$

$$d = \sqrt{x^2 - 3x + 4}$$

$$\frac{dd}{dt} = \frac{2x-3}{2\sqrt{x^2-3x+4}}\frac{dx}{dt} \qquad = \frac{2(3)-3}{2\sqrt{(3)^2 - 3(3) + 4}}4 = 3$$

(b) $$\tan\theta = \frac{\sqrt{x}}{(x-2)} \qquad \sec^2\theta\frac{d\theta}{dt} = \frac{-1}{\sqrt{x}(x-2)^2}\frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \frac{-\cos^2\theta}{\sqrt{x}(x-2)^2}\frac{dx}{dt} \quad \frac{d\theta}{dt} = \frac{-1/4}{\sqrt{3}(3-2)^2}4 = -\frac{\sqrt{3}}{3}$$



Pbm 41: A particle is moving along the curve $y = x/(x^2+1)$. Find all values of x at which the rate of change of x with respect to time is three times that of y.

$$\frac{dy}{dt} = \frac{1}{3}\frac{dx}{dt} \qquad \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1}{3} = \frac{dy}{dx}$$

$$y = \frac{x}{\left(x^2+1\right)}$$

$$\frac{dy}{dx} = \frac{\left(x^2+1\right) - x\left[2x\right]}{\left(x^2+1\right)^2} = \frac{1-x^2}{\left(x^2+1\right)^2} = \frac{1}{3}$$

$$3 - 3x^2 = \left(x^2+1\right)^2$$

$$x^4 + 5x^2 - 2 = 0 \qquad x^2 = .3722813$$

$$x = \pm.61015 \qquad y = \pm.444624$$

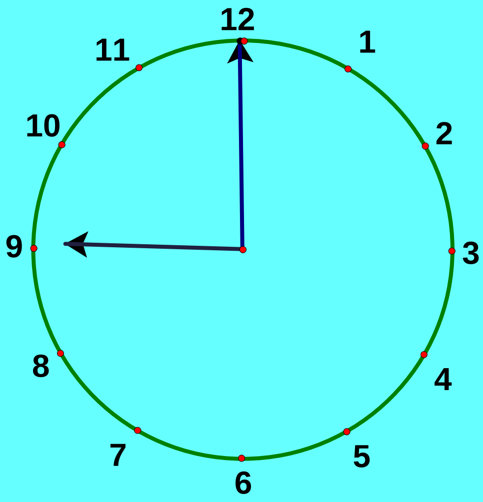
PBM 43: The thin lens equation in physics is 1/s + 1/S = 1/f where s is the object distance from the lens, S is the image distance from the lens, and f is the focal length of the lens. Suppose that a certain lens has a focal length of 6 cm and that an object is moving toward the lens at the rate of 2 cm/sec. How fast is the image distance changing at the instant when the object is 10 cm from the lens? Is the image moving away from the lens or toward the lens.

$$\frac{1}{s}+\frac{1}{S}=\frac{1}{f}$$

$$-\frac{1}{s^2}\frac{ds}{dt}-\frac{1}{S^2}\frac{dS}{dt}=0$$

$$\frac{ds}{dt}=-\frac{s^2}{S^2}\frac{dS}{dt}=-\frac{15^2}{10^2}(-2)=4.5\frac{cm}{sec}$$

Away



PBM 46:On a certain clock the minute hand is 4 in long and the hour hand is 3 in. long. How fast is the distance between the tips of the hands changing at 9 o'clock?

$$d^2 = 4^2 + 3^2 - 2(4)(3)CosD$$

$$\frac{dd}{dt} = \frac{12Sin\theta}{d}\frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = \frac{2\pi}{60} - \frac{2\pi}{360} = \frac{11\pi}{360}\frac{rad}{\min}$$

$$\frac{dd}{dt} = \frac{12(1)}{5}\frac{11\pi}{360} = \frac{11\pi}{150}$$